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# Methodology enhancement of resonant ultrasound spectroscopy (RUS) to identify effective elastic properties of an aluminum sample

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**Abstract.** This work aims to enhance the methodology for characterizing the elastic properties of massive sample. A novel methodological approach is coupled to the resonant ultrasonic spectroscopy (RUS). The innovation consists in a *find-peaks* function used to automatically extract resonant frequencies from the mechanical frequency response of the sample, and to a *K-means* clustering algorithm used to pool the data obtained from multiple spectra measured on the same sample. It results in a fully automated detection of all predicted resonant mode of a specimen and hence increase both ease of use and accuracy of the RUS method. The acoustic resonance measurements are carried out using two shear transducers having a center frequency at 2.25 MHz. In view to solve the inverse problem, two hypotheses are implemented, either isotropic or orthotropic. The obtained results show the validity of the proposed methodology for both assumptions and are in agreement with the literature.

**Keywords:** Resonant Ultrasound Spectroscopy (RUS), stiffness coefficients, Aluminum elastic properties, Non-Destructive Evaluation (NDE), inverse problem solution.

# 1 Introduction

The metallic materials are composed of polycrystalline grains which effective properties usually results in a supposed effectively isotropic medium. Nevertheless, many exhibit a degree of anisotropy depending on the thermomechanical fabrication process, which may induce a specific distribution arrangement of crystallographic orientations of the grains in the aggregate, such as grain orientation along the extrusion or lamination axis, or grain elongation along the rolling direction (Hutchinson 2015). Classical mechanical testing methods (such as tensile, bending, compression tests) are destructive and therefore necessitates many sample tests to characterize the elastic properties of a given composition. Moreover, the characterization result is depending on the testing conditions, such as strain rate. As an alternative methodology, characterization techniques based on acoustic methods give an information on the bulk properties of a sample along the wave propagation direction (Kim, Sachse, and Every 1993). As a generalization of the thickness resonance characterization, the resonant ultrasound spectroscopy (RUS) constitutes a non-destructive evaluation (NDE) method where the mechanical resonance frequencies of a specimen are evaluated and used to determine the elastic properties of a material.

The RUS method, as extensively described by (Migliori and Sarrao 1997) is used to identify the elastic properties of a sample having a canonical geometry. It is based on harmonic stimulation in a range of frequencies that produces ultrasonic resonance patterns as a result by natural vibration of the sample. The RUS method is a dynamic methodology based on the analysis of the resonance frequencies provides an elastic characterization of the bulk material. (Demarest 1971) show the interest in using cubes with this technique. This study showed the limits of identifying specific vibration modes in highly anisotropic crystals. The proposed identification method is not simple and accurate enough such as in isotropic or cubic case of study. The limits are also linked to the accuracy and reliability of the experimental conditions, such as the precision of experimental measurements and the consistency in determining elastic constants from resonance frequencies.

This method was demonstrated to be equally successful for vibration analysis of cylinders and rectangular parallelepipeds by (Holland 1968). Also, (Ohno *et al.* 1986) used the fundamental Rayleigh-Ritz technique to apply RUS to spherical geometry. Furthermore, this approach is used to non-isotropic single-crystal samples (Ohno *et al.* 1986). This technique has been expanded upon by other research to include samples with intricate geometries, and it has been used with piezoelectric materials like quartz (Ogi *et al.* 2006). The synchronization of resonance frequency detection for homogeneous rocks, *i.e.* uniform in composition and structure throughout the sample, was proved by (Zadler *et al.* 2004), emphasizing the quality factor  $Q$ , which is related to resonance peak width. Anisotropic metallic samples

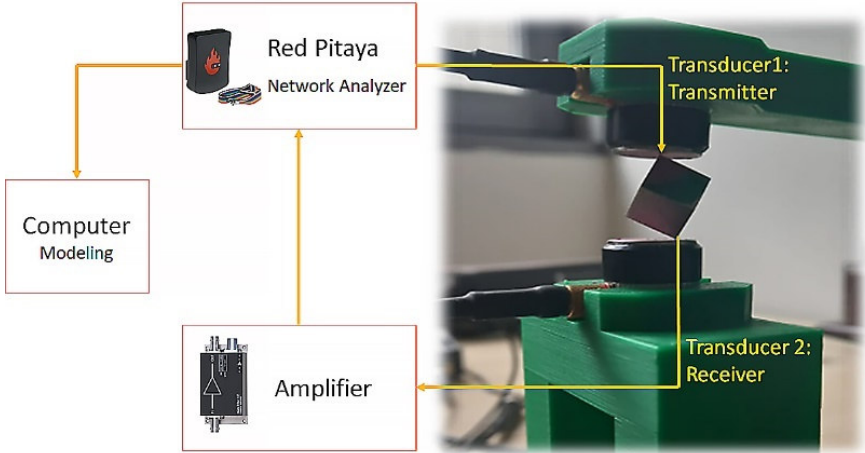
with a polycrystalline texture are evaluated using RUS as a non-destructive technique by (Evans *et al.* 2021).

(Bernard, Grimal, and Laugier 2014) employed the RUS technique to evaluate mechanical characterization of viscoelastic samples made of polymers, in which acoustic waves are rapidly attenuated and mechanical resonances are strongly damped. As an illustration, (Bernard, Grimal, and Laugier 2013) have provided evidence that RUS may be effectively applied to damping materials in the medical domain, by taking a small cubic bone structure as sample. As a result, the inverse problem for a solid rectangular parallelepiped shape to figuring out their stiffness of anisotropy. Besides, they are validating the Rayleigh-Ritz method to predict resonant frequencies into RUS method, taking account the mass, stiffness coefficient, and geometry of the sample material. The stiffness coefficients  $C_{ij}$  and the eigenvector containing the resonant frequencies, i.e.  $f_{res} = g(C_{ij})$ , can be provided by a numerical resolution. For a rectangular parallelepiped sample, the numerical solution relies on a decomposition of the displacement field on a basis of polynomial functions, and reduce to an eigenvalue problem. A fact about the resolution of RUS measurement is typically focused on a limited frequency range, with a few resonant frequencies. The difficult challenge of this method is to easily associate the measured frequencies with the predicted ones. In addition, limitations related to damping effects make it complex the signal processing for pairing predicted and measured resonant frequencies. Consequently, a Bayesian formulation is proposed by (Bernard *et al.* 2015) that automatically lowers the ambiguities of the inverse problem. Thus, the Markov-Chain-Monte-Carlo (MCMC) approach (Gibbs-sampling) is used to sample the joint posterior distribution of mappings and stiffness. This process begins with a preliminary parameter estimate and continues iteratively until the algorithm converges satisfactorily. Nevertheless, when applied to damping materials, many predicted resonant modes may not be observed experimentally and ambiguities in pairing measured and predicted frequencies can arise.

This paper includes an integration of the RUS methodology, in order to identify elastic properties by means of a numerical function related to the literature investigations. The novelty of this work consists in the articulation of two recent advances, i.e. *find-peaks* function and *K-means* clustering method. The combination of those methods helps to solve the presented limitations such as the reliability and validity of experimental data, by avoiding false resonant frequencies and pairing between theoretical and experimental resonant frequencies. The identification of the experimental measurements by involving *find-peaks* function and *K-means* clustering method present a promising approach to be crucial for expanding to damping material such as polymers and composites. The presented application case of study is focused on the elastic characterization of a massive aluminum (Alu) sample. The obtained adjustment results and their accuracy based on the isotropic and orthotropic assumptions are discussed.

## 2 Experimental setup and proposed analysis enhancement

### 2.1 Setup and experiment



**Fig. 1. RUS setup and Alu specimen inserted in between the two transducers**

The material specimen used in this study is a massive Alu rectangular parallelepiped, whose dimensions are indicated in Table 1. The measurement setup is largely inspired by (Balakirev *et al.* 2019). It consists of a pair of shear ultrasonic transducers (2.25 MHz central frequency), a 3D-printed support (green part in Fig. 1), a charge amplifier (QUARTZ TUNING FORK-HQA-15M-10T), and a Red Pitaya board acting as a network analyzer. The purpose of this setup is to record the mechanical transfer function of the sample. To this purpose, a frequency scan is performed by the Red Pitaya board, which acts both as a signal generator and recorder over a wide frequency range. One of the transducers is used as an emitter and generates mechanical vibrations inside the sample, while the second one is used as a receiver and convert mechanical vibration to an electrical signal.

**Table 1. Geometry and mass of the sample selected.**

Material	Alu
Length (mm)	12.06
Geometry Width (mm)	10.40
Height (mm)	8.08
Density (kg/m <sup>3</sup> )	2782

As illustrated in Fig. 1, the sample is placed a pair of opposite corners between the transducers. This is for two reasons: first, it reduces stress contact, so that the measured resonant frequencies are close to stress-free boundary condition resonances; second, the corners are low symmetry points which are never nodes of the resonant modes, increasing the number of measured frequencies. Due to the limited 1 V output of the board, a charge amplifier is used in reception to improve signal-to-noise ratio. Furthermore, the acquisition is repeated several times with different position of the sample to get most possible resonant modes.

A LabView user interface, developed to control the Red Pitaya specifically for RUS experiments (Balakirev *et al.* 2019) was used. It allows setting of experiment parameters such as minimum frequency, maximum frequency and frequency step. The interface and a recorded resonance spectrum are pictured in Fig. 2. Sharp peaks are visible at the resonance frequencies of the specimen. After the frequency sweep is complete, the resonance spectrum is captured and analyzed using an external program to fit an elastic model of the sample being studied.

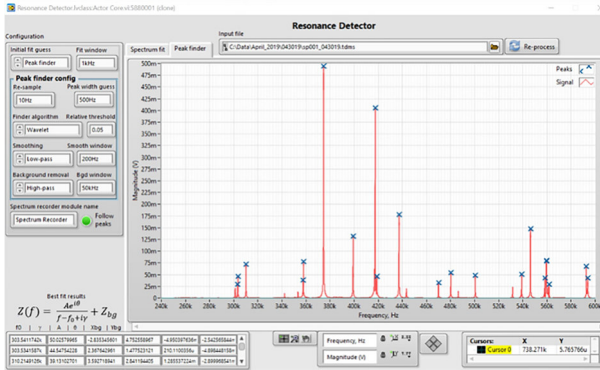


Fig. 2. LabView program developed by (Balakirev *et al.* 2019).

## 2.2 RUS data processing and analysis

With the RUS method, the specimen's elastic properties  $C_{ij}$  (components of the stiffness matrix) are estimated through an inverse problem. The stress-free boundary mechanical resonant frequencies obtained as described above are compared to model-predicted frequencies, and the stiffness coefficients  $C_{ij}$  are adjusted until the best match between experimental and model-predicted frequencies is obtained.

Mathematically, one seeks to minimize the  $F(C_{ij})$  cost function defined as:

$$F(C_{ij}) = \sum_{k=1}^{N_{modes}} \left| \frac{f_{k,Experiment} - f_{k,Model}(C_{ij})}{f_{k,Experiment}} \right| \quad (1)$$

Indeed, given shape of the specimen, dimensions and mass density, its free-resonant frequencies can be predicted with high accuracy for a given set of elastic coefficients using the so-called Rayleigh-Ritz analytical method (Migliori and Sarrao 1997). The  $F(C_{ij})$  cost function (eq. (1)) can then be minimized iteratively.

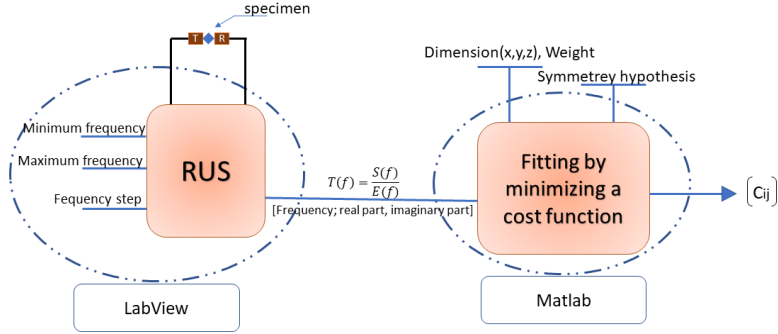


Fig. 3. RUS and acquisition overview to identify elastic constants.

### 2.3 Proposed methodology enhancement

A difficulty that arises in RUS data analysis is that, for each acquisition, not all the resonant frequencies are detected in the spectrum (Bernard *et al.* 2015). Pairing of the experimental and model-predicted frequencies, necessary to build the cost function (1) is therefore non-trivial. This can prevent convergence of the inversion and accurate estimation of the elastic coefficient.

A solution is to record multiple spectra for the same specimen placed in different positions in-between the transducers, increasing the chances to record all of the predicted frequencies. This however entails two difficulties: first, there are a lot of data to process, and manually picking the resonant frequencies in each spectrum is tedious and prone to errors; second, one must find a way to effectively pool the frequencies obtained from multiple experiments, not knowing exactly whose frequencies are present, or not, in each measurement. The enhancement introduced in this work aims at solving these two challenges.

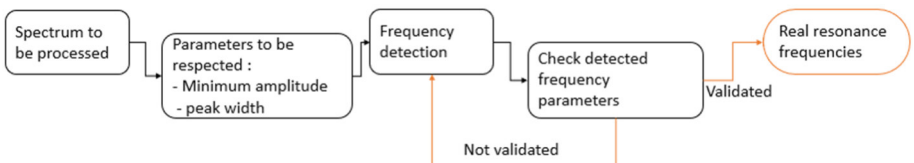


Fig. 4. Find-peaks function process.

A peak-finding method was used to automatically peak the resonant frequencies in each spectrum. We used the *findpeaks* function available in Matlab. Its process is described in Fig. 4. It takes multiple parameters such as minimum amplitude and minimum spacing between the frequencies to prevent picking peaks that are too wide or too narrow, which may be caused by noise. All the frequencies peaked in the multiple spectra are then pooled together, and a clustering algorithm is used to group frequencies by proximity (frequencies corresponding to the same resonant mode recorded in different spectra are very close together) and provide an average and robust value for each resonant frequency. The *K-means* algorithm was implemented. *K-means* clustering function based on regrouping frequencies data into independent similar clusters, based on proximity, and iteratively reposition the cluster center in order to minimize the distance within each cluster.

### 3 Results and discussion

#### 3.1 Isotropic inverse problem

Within this research, 9 experimental acquisitions for the Alu sample are used as input data in view to identifying the significant modes. The inverse problem was distributed in the isotropic hypothesis, taking into account the first points of elastic coefficient to start the theoretical adjustment with the experimental ones. Thus, the isotropic stiffness components have been initialized with  $C_{44} = 27.5$  GPa and  $C_{11} = 110$  GPa. For the isotropic case, this initial set of values  $\{C_{11}; C_{44}\}$  is selected from a preliminary visual adjustment using a Graphical User Interface (GUI) developed in Matlab and illustrated in Fig. 5. This GUI allows to adjust the elastic coefficient and visualize the superposition of the theoretical resonant frequencies over the measured spectrum.

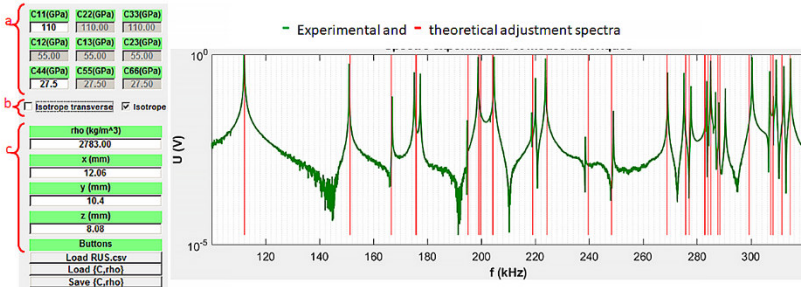


Fig. 5. Isotropic adjustment with << RUS\_UI\_Cij >> interface: (a) stiffness coefficients  $C_{ij}$ ; (b) symmetry hypothesis; (c) specimen parameters.

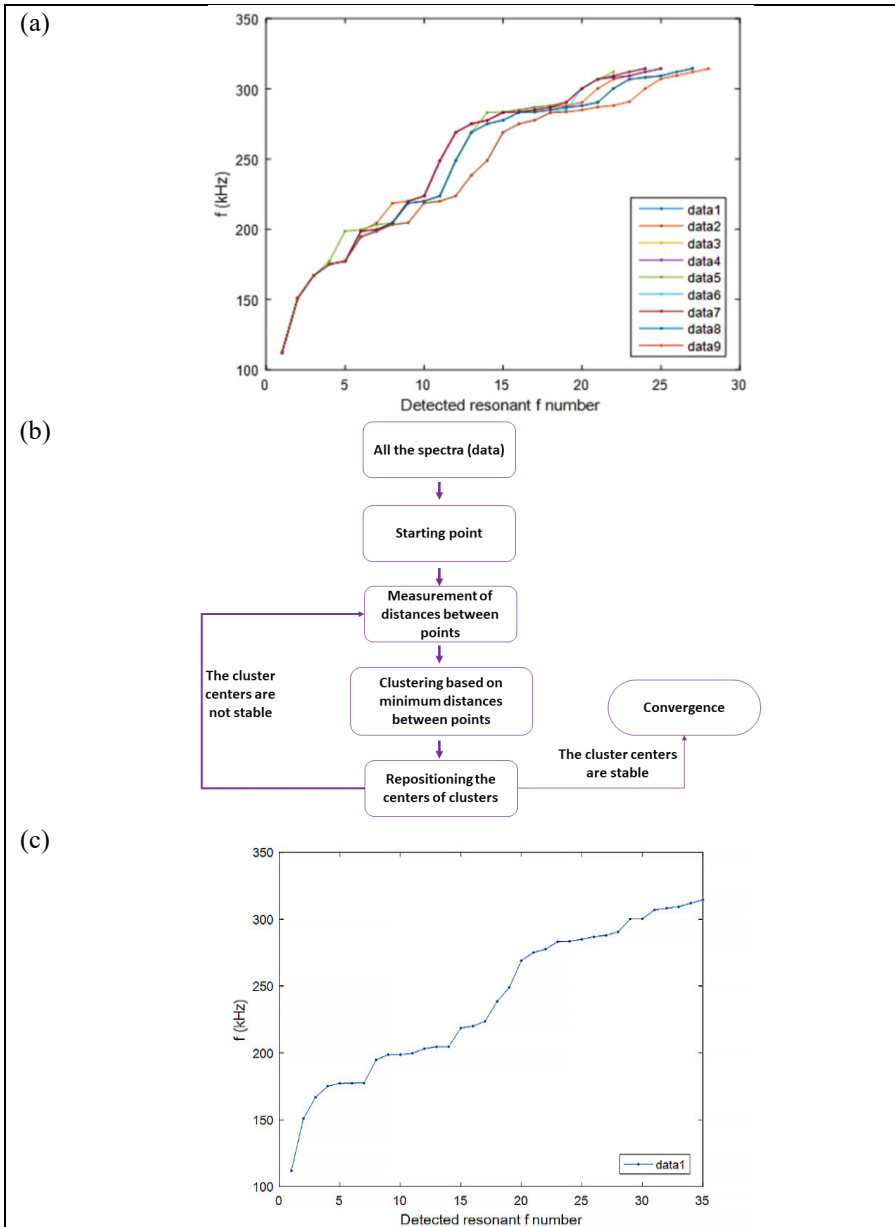


Fig. 6. Clustering of spectrum data: (a) *Find-peaks* and (b-c) *K-means* adjustment process.

According to (Bernard *et al.* 2015), the stiffness matrix is to be estimated throughout successive trials and error estimation until convergence. For coefficients  $C_{11}$  and  $C_{44}$ , *a posteriori* mean values and standard deviations are used as estimates of stiffness constants and uncertainties respectively. In the tables below,  $C_{ij}$  values are representing the maximum value estimates in the distribution, while  $\sigma(C_{ij})$  are representing the standard deviations of the distribution.

**Table 2. Solutions of the isotropic inverse problem of the Alu specimen without clustering.**

Acquisition	$C_{11}$ (GPa)	$\sigma(C_{11})$ (GPa)	$C_{44}$ (GPa)	$\sigma(C_{44})$ (GPa)	$RMSE$ (%)
1	114.4865	$\pm 2.9834$	27.4408	$\pm 0.0651$	0.32
2	114.6164	$\pm 2.8352$	27.4316	$\pm 0.0612$	0.34
3	114.4852	$\pm 3.1649$	27.4358	$\pm 0.0697$	0.32
4	114.0186	$\pm 2.4877$	27.4497	$\pm 0.0588$	0.33
5	113.3123	$\pm 2.6075$	27.4622	$\pm 0.0647$	0.37
6	111.8977	$\pm 3.1739$	27.4249	$\pm 0.0837$	0.33
7	113.5397	$\pm 3.0028$	27.4390	$\pm 0.0652$	0.33
8	114.5555	$\pm 2.6770$	27.4349	$\pm 0.0647$	0.32
9	114.8971	$\pm 2.8265$	27.4329	$\pm 0.0605$	0.37
Average	113.9788	$\pm 2.8621$	27.4391	$\pm 0.0660$	0.34

As reported in Table 2, the results of the isotropic inverse problem for each acquisition, show a relatively good reproducibility. It can be seen that the Root Mean Square Error ( $RMSE$ ) do not exceed 0.37%, which indicates a very stable solution, and the uncertainty can be considered negligible. Those results are almost similar to the acquisition using the resonance clustering method to adjust the number of distinct resonance frequencies. As illustrated in Fig.6, the clustering method results in an average spectrum containing 30 resonance frequencies. Table 3 summarizes the obtained results, with a slightly lower evaluation of the  $RMSE$ , evolving from 0.37% without clustering, decrease to 0.33% with clustering. Those low  $RMSE$  values support the hypothesis of an isotropic symmetry of the material, since a model based on this hypothesis is able to accurately predict the value of the 30 first resonant frequencies of the sample.

**Table 3. RUS estimation of the isotropic parameters by solving the inverse problem based on the spectrum resulting from the *find-peaks* and *K-means* functions.**

$C_{11}$ (GPa)	$\sigma(C_{11})$ (GPa)	$C_{44}$ (GPa)	$\sigma(C_{44})$ (GPa)	$RMSE$ (%)
114.7863	$\pm 2.5827$	27.4095	$\pm 0.0568$	0.33

### 3.1 Orthotropic inverse problem

For the orthotropic inverse problem hypothesis, the starting points of the elastic coefficients necessitates an initialized vector of values. Thus, the  $C_{ij}$  (GPa) are initialized with:  $\{C_{11}; C_{22}; C_{33}; C_{12}; C_{13}; C_{23}; C_{44}; C_{55}; C_{66}\} = \{110; 110; 110; 55; 55; 55; 27.5; 27.5; 27.5\}$  GPa. As for the isotropic hypothesis, these initial values are selected utilizing the same methodology as depicted in the case of isotropic inverse problem. As summarized in Table 4, the final adjustment shows that the orthotropic assumption leads to an averaged error evaluated through the Root Mean Square Error,  $RMSE = 0.21\%$  lower than the  $0.5\%$  noted by (Bernard, Grimal, and Laugier 2013). This result justifies the orthotropic hypothesis, but need further discussion.

**Table 4. Solutions of the orthotropic inverse problem of the Alu specimen based on the preprocessed *K-means* spectrum.**

	$C_{11}$ (GPa)	$C_{22}$ (GPa)	$C_{33}$ (GPa)	$C_{12}$ (GPa)	$C_{13}$ (GPa)	$C_{23}$ (GPa)	$C_{44}$ (GPa)	$C_{55}$ (GPa)	$C_{66}$ (GPa)
$C_{ij}$	114.1251	112.2254	111.3610	58.7814	57.2072	57.0969	27.2509	27.4967	27.7680
$\sigma(C_{ij})$	$\pm 1.9538$	$\pm 2.1284$	$\pm 2.5130$	$\pm 2.0837$	$\pm 2.4625$	$\pm 2.2563$	$\pm 0.1123$	$\pm 0.1374$	$\pm 0.1263$
	$RMSE$ (%)					0.21			

As a comparison between the isotropic and orthotropic hypothesis the effective engineering moduli of the Alu sample, *i.e.* Young's modulus and Poisson's coefficient  $\{E, \nu\}$  are evaluated along the direction 1, using the following equations:

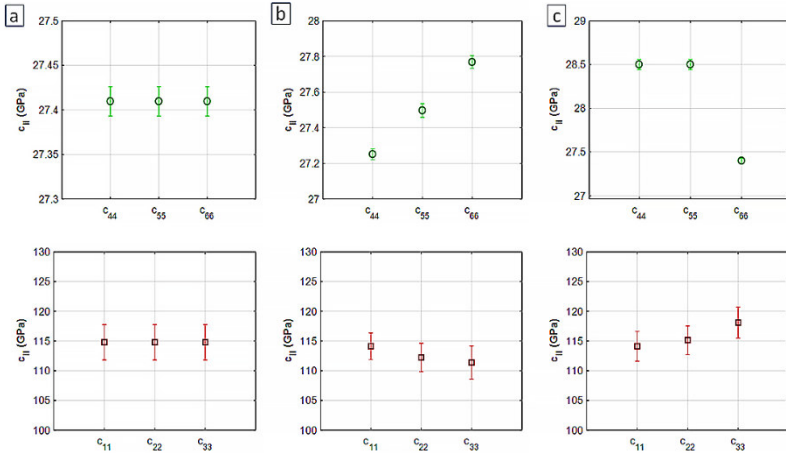
$$\begin{cases} E = C_{11} - 2 \frac{C_{12}^2}{C_{11} + C_{12}} \\ \nu = \frac{C_{12}}{C_{11} + C_{12}} \end{cases} \quad (2)$$

The resulting effective engineering moduli  $\{E, \nu\}$  of the Alu sample are summarized in Table 5.

**Table 5. Effective engineering moduli in the isotropic and orthotropic hypothesis.**

	$E$ (GPa)	$\nu$
Isotropic	73.63	0.34
Orthotropic	74.16	0.34

The  $C_{ij}$  obtained for both symmetry hypothesis (isotropic and orthotropic) are compared with those of (Le Bourdais *et al.* 2022). The adjustment process of the later for the determination of the elastic coefficients is based on the Bayesian orthotropic RUS inversion. From an isotropic approximation, the results of (Le Bourdais *et al.* 2022) lead to a Young's modulus of 75 GPa, and a Poisson's ratio of 0.34. In our case, the distribution of stiffness coefficients is well obtained in both hypotheses, with a negligible error. As illustrated in Fig. 7, with scales adapted to compare elastic shear constants, it can be observed that the represented deviations are clearly readable.



**Fig. 7. Comparison of  $C_{ij}$  according to the isotropic (a), orthotropic (b) hypothesis, and with the results of (Le Bourdais *et al.* 2022) (c), according to the orthotropic hypothesis.**

To evaluate the anisotropy of the sample, the Zener factor  $A$ , as used by (Evans *et al.* 2021), is defined by the equation (3). The more the Zener factor  $A$  approaches to 1, the more material is isotropic.

$$A = \frac{2C_{44}}{C_{11} - C_{12}} \quad (3)$$

Following the orthotropic hypothesis, from Table 5, the Zener factor is evaluated by  $A = 0.967$ . Thus, the hypothesis of isotropy of the massive aluminum sample is confirmed. Furthermore, this global Zener factor does not make any distinctions between the stiffness coefficient types. Thus equation (4), as defined by (Evans *et al.* 2021; Thomsen 1986), gives additional parameters to evaluate the anisotropy type and rate. The triplet  $\{\varepsilon, \gamma, \delta\}$  gives a more nuanced estimate. Among this triplet values, the first parameter expresses the variability of the compression constants, the second expresses the variability of the shear constants, and last is the expansion anisotropy factor.

$$\begin{cases} \varepsilon = \frac{C_{11}-C_{33}}{2C_{33}} \\ \gamma = \frac{C_{66}-C_{44}}{2C_{44}} \\ \delta = \frac{(C_{13}+C_{44})^2-(C_{33}-C_{44})^2}{2C_{33}(C_{33}-C_{44})} \end{cases} \quad (4)$$

According to equation (4), the anisotropy parameters triplet  $\{\varepsilon, \gamma, \delta\}$  gives an estimate of the relative distance to the isotropic hypothesis. Thus, the anisotropy of compression was evaluated very low with  $\varepsilon = 1.24\%$ , the anisotropy of the shear parameters was negligible with  $\gamma = 0.95\%$  and anisotropic expansion was estimated at  $\delta = 0.31\%$ . However, the average anisotropy factor is  $0.83\%$ , and does not exceed  $1\%$ : the studied massive Alu sample can be qualified as isotropic.

## 4 Conclusion

In this paper, the resonant ultrasound spectroscopy (RUS) was investigated as a non-destructive evaluation (NDE) tool to quantify the elastic properties of a canonical sample. Taking an aluminum sample as object of study, the stiffness coefficients have been successfully evaluated by means of two shear transducers having a 2.25 MHz center frequency. More specifically, the enhanced adjustment procedure for the inverse problem has provided a significant contribution to identify the elastic properties following either the isotropic or orthotropic hypothesis. Thanks to the preprocessing of the resonance spectra through the use of *find-peaks* and *K-means* functions, a significant enhancement of the adjustment procedure has been brought. This methodology could be applied in the future research to polymers, which presents in addition the damping problem. Overall, our study improved the understanding of the stiffness components, depending on the isotropic or orthotropic hypothesis, and their accurate evaluation, thus opening the doors for other research in material science and engineering applications. We foresee that this extended RUS

methodology, improved both regarding accuracy and ease-of-use, could be used for systematic study of texture in large collections of polycrystalline metallic samples, including samples obtained from additive manufacturing. In addition, in future research we intend to extend the proposed methodology to polymers. In this case we will face the additional challenge of high viscoelastic damping which causes broad and weak resonant peaks.

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## List of Abbreviations

RUS:	Resonant Ultrasound Spectroscopy
NDE:	Non-Destructive Evaluation
Alu:	Aluminum
$C_{ij}$ :	Stiffness matrix coefficients
MCMC:	Markov-Chain-Monte-Carlo
<i>RMSE</i> :	Root Mean Square Error

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