



**HAL**  
open science

## Control systems with paraboloid nonholonomic constraints

Timothée Schmoderer

► **To cite this version:**

Timothée Schmoderer. Control systems with paraboloid nonholonomic constraints. Workshop on "Optimal Control Theory", Jun 2022, Rouen, France. hal-03704279

**HAL Id: hal-03704279**

**<https://normandie-univ.hal.science/hal-03704279>**

Submitted on 24 Jun 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Control systems with paraboloid nonholonomic constraints

TIMOTHÉE SCHMODERER

*INSA Rouen Normandie, Rouen, France*

## ABSTRACT

In this talk, we present a characterisation and a classification of control systems (both control-affine and fully nonlinear) whose field of admissible velocities forms a paraboloid submanifold in each tangent space. We thus continue previous works on affine and linear Pfaffian equations whose admissible velocities form either an affine or a linear subspace in each tangent space; see [4, 5, 1, 6]. Precisely, we consider a smooth  $n$ -dimensional manifold  $\mathcal{X}$  equipped with local coordinates  $x$  and we call a nonholonomic constraint *paraboloid* if it is given by an equation of the form

$$S_Q(x, \dot{x}) = \dot{z} - \dot{y}^t Q(x) \dot{y} - b(x) \dot{y} - c(x) = 0,$$

where  $x = (z, y)$ , with  $y = (y_1, \dots, y_{n-1})$ ,  $Q$  is a smooth symmetric matrix of full rank,  $b$  is a smooth covector, and  $c$  is a smooth function. The case  $n = 2$  corresponds to  $S_Q$  describing a parabola  $\dot{z} = a(x) \dot{y}^2 + b(x) \dot{y} + c(x)$  in each fiber  $T_x \mathcal{X}$  and is treated (and generalised to elliptic and hyperbolic cases) in [3] and is extended to all dimensions by the speaker in his thesis [2].

We will introduce a novel class of control-affine systems whose trajectories satisfy the paraboloid nonholonomic constraint given by  $S_Q$  and we give a characterisation of that class among all control-affine systems. Next, we will classify paraboloid control systems under feedback transformations. In particular, we will insist on the differences between the cases  $n = 2$  and  $n \geq 3$ . Moreover, we will show how our classification of control systems provides a corresponding classification of paraboloid equations  $S_Q = 0$ .

This is a joint work with *Witold Respondek* from *INSA Rouen Normandie*.

## References

- [1] Bronislaw Jakubczyk and Witold Respondek. “Feedback Equivalence of Planar Systems and Stabilizability”. *Robust Control of Linear Systems and Nonlinear Control*. Springer, 1990.
- [2] Timothée Schmoderer. “Study of Control Systems under Quadratic Non-holonomic Constraints. Motion Planning, Introduction to the Regularised Continuation Method.” PhD thesis. INSA Rouen Normandie, 2022. URL: <http://theses.fr/s221665>.
- [3] Timothée Schmoderer and Witold Respondek. “Conic Nonholonomic Constraints on Surfaces and Control Systems”. *arXiv:2106.08635 [math]* (2021). URL: <http://arxiv.org/abs/2106.08635>.
- [4] Michail Zhitomirskii. *Typical Singularities of Differential 1-Forms and Pfaffian Equations*. Vol. 113. American Mathematical Soc., 1992.
- [5] Michail Zhitomirskii. “Singularities and Normal Forms of Smooth Distributions”. *Banach Center Publications* 32.1 (1995). DOI: [10.4064/-32-1-395-409](https://doi.org/10.4064/-32-1-395-409).
- [6] Michail Zhitomirskii and Witold Respondek. “Simple Germs of Corank One Affine Distributions”. *Banach Center Publications* 44.1 (1998). DOI: [10.4064/-44-1-269-276](https://doi.org/10.4064/-44-1-269-276).