



HAL
open science

Tunable multidispersive bands of inductive origin in piezoelectric phononic plates

K. Mekrache, R. Sainidou, P. Rembert, N. Stefanou, Bruno Morvan

► **To cite this version:**

K. Mekrache, R. Sainidou, P. Rembert, N. Stefanou, Bruno Morvan. Tunable multidispersive bands of inductive origin in piezoelectric phononic plates. *Journal of Applied Physics*, 2021, 130 (19), pp.195106. 10.1063/5.0065184 . hal-03479298

HAL Id: hal-03479298

<https://normandie-univ.hal.science/hal-03479298>

Submitted on 14 Dec 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Copyright

Tunable multidispersive bands of inductive origin in piezoelectric phononic plates

K. Mekrache,¹ R. Sainidou,¹ P. Rembert,^{1, a)} N. Stefanou,² and B. Morvan¹

¹⁾*Laboratoire Ondes et Milieux Complexes UMR CNRS 6294, UNIHAVRE, Normandie University, 75 rue Bellot, 76600 Le Havre, France*

²⁾*Section of Condensed Matter Physics, National and Kapodistrian University of Athens, Panepistimioupolis, GR-157 84 Athens, Greece*

(Dated: 18 October 2021)

A variety of multidispersive, localized or extended in frequency, bands, induced by inductance-based external electric circuits in piezoelectric phononic plates, is studied both theoretically and experimentally in this work. Their origin, tightly related to an equivalent LC -circuit behavior is analyzed in detail and their interaction with the Lamb-like guided modes of the plate is also discussed. These bands, easily tuned by the choice of the parameters of the external electric circuitry lead to a non-destructive, real-time control of the dispersion characteristics of these structures. Our device and analysis can find application in improvement of surface acoustic wave (SAW) components, by offering additional degrees of freedom.

1 I. INTRODUCTION

Controlling the dispersion of elastic waves has become a key issue when dealing with sound mitigation, vibration energy harvesting and, more generally, applications in which elastic waves have to be guided, collimated or trapped. The use of metamaterials has been an important step towards the feasibility of this control¹⁻³ by shaping the dispersion relation of these complex media. Several approaches have been proposed, the most common being the periodic structuration of the medium (phononic crystal) leading to folding of its frequency bands and opening up of band gaps. To tailor the band structure at subwavelength regime, a very promising means has been the use of the locally-resonant materials that produce strong dispersive effects related to negative effective bulk modulus and mass density⁴⁻⁶. These structures suffer, however, from an intrinsic narrow frequency range behavior of the resonant phenomena. To overcome this limitation, a more sophisticated structuring has been recently proposed, facilitated by the use of new manufacturing techniques, such as 3D printing. The concepts of rainbow metamaterials, gradient index and hierarchical metamaterials allowing to model the dispersion of the waves on a broader frequency range have emerged⁷⁻⁹.

However all these attractive properties of metamaterials, even in the case of wide-range frequency phenomena, will stay less appealing, especially for application purposes, if the question of frequency tunability in their response is not answered. For a fixed geometrical arrangement of the constituents of the complex medium, the resulting frequency response cannot be adapted in real time in a real device. A simple way to introduce this frequency tunability/agility lies in the use of active components whose control can be achieved with the help

of an externally applied field (magnetic^{10,11}, electric¹², thermal^{13,14}, and radiative^{15,16}). The mechanical properties (mass density, elastic moduli) of the active component are thus modified, leading to reversible changes in its frequency band structure. Among others, piezoelectric-based structures have been shown to be excellent candidates for the real-time control of the propagation of elastic waves. The electro-mechanical coupling, naturally present in these materials, offers a versatile tool to shape the effective elastic properties of the metamaterial, through a variety of electrically controlled external conditions applied at the structure¹⁷⁻²⁶.

In this paper, we use a piezoelectric material as a substrate (plate) on which metallic strips (electrodes) are structured. In previous works²³⁻²⁶, some of us have shown that propagation of Lamb-like guided modes in these phononic crystal piezoceramic plates exhibits an interesting tunable character easily controlled by external electric circuits through appropriate electric boundary conditions (EBCs) applied at the metallic strips. In particular, the band structure of the plate is significantly modulated by the presence of electric resonant modes originating from the external inductance-loads coupled to the effective capacitive behavior of the piezoelectric plate. Some basic methodological tools of our analysis are already presented in Ref. 26. Here, we extend them in order to give a synthetic panel that these systems can offer. We review under a new light some of the possibilities to mould the dispersion characteristics of tunable localized bands originating from an equivalent LC -circuit behavior, inherent in piezoelectric phononic plates, when loaded with inductance external circuits. We focus on three typical representative cases including external L -based loads that generate, each of them, a variety of multidispersive, tunable bands of electromagnetic (EM) origin, easily controlled via the external circuitry parameters (the inductance, in our case). These cases, first given in a qualitative schematic manner, cover, respectively, low, intermediate and higher frequency regions in a typical dispersion plot. Moreover, compared to previ-

^{a)}Electronic mail: pascal.rembert@univ-lehavre.fr

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184

ous works^{23–26}, an important improvement of our experimental device—especially its electronic part—is made, towards a wider range of choices in real-time control and automatization, combined to a robust and promising signal processing technique leading to a finer analysis of the experimental data. Usually, surface acoustic wave (SAW) devices offer only input and output ports, restricting the analysis exclusively in the frequency domain. Our device, allows to probe the surface waves along their path, thus offering the wavevector projection along this path as an additional degrees of freedom, which could provide an improvement in the domain of SAW components.

The paper is organized as follows. In Sec. II we present the general characteristics of the materials and structures considered in this study, and provide all necessary details concerning the experimental setup employed for our measurements, as well as the methods and techniques used in the analysis of the experimental data and in the numerical calculations, that follow in Sec. III. For the different structures under study, the analysis of their dispersion properties, strongly influenced by the external loads, in terms of underlying physical mechanisms responsible for these effects, as well as in comparison with their experimental observation, constitute the main part of Sec. III, before concluding the paper in Sec. IV.

II. MATERIALS AND METHODS

A. Sample structure

We shall be concerned, in this study, with thin homogeneous piezoceramic plates whose both surfaces are decorated with periodic one-dimensional (1D) arrays (lattice constant a_0) of parallel and face-to-face metallic strips of width w and negligible thickness. A schematic representation of the structure is given in Fig. 1(a). The plate is made of PZT (Navy VI) and in all theoretical calculations presented in this paper it will be considered to be infinite along x_1 - and x_2 -directions that coincide with the transversely isotropic plane of the piezoceramic material; the x_3 -axis coincides with the polarization axis of the material.

The sample used in the experiments, is a finite square-shaped plate (edge length 48 mm and thickness $h = 1.6$ mm), as can be seen in Fig. 1(b), on both sides of which silver strips (rectangular electrodes) have been deposited periodically; their thickness is of the order of 15 μm and will be neglected in what follows. In the real structure, the lattice period dictated by the metallic strips (width $w = 0.7$ mm, separation gap $s = 0.3$ mm) equals $a_0 = s + w = 1$ mm, but as we have already explained in previous works^{24–26} the true lattice constant of the crystal a can be non-destructively extended to be a multiple of a_0 , controlled through the application of ap-

propriate EBCs at each pair of (up- and down-side) electrodes. The latter is connected in the most general case to external electric circuits of characteristic impedance Z^u and Z^d , as schematically depicted in Fig. 1(c). In this manner, the elementary blocks of length $a_0 = w + s$ may have identical ($a = a_0$) or different ($a = \kappa a_0$, $\kappa = 2, 3, \dots$) EBCs throughout the structure, thus tuning at will the real periodicity of the crystal. The 1D Brillouin zone (BZ) of the crystal is accordingly shortened; we will denote its center ($k_1 a / \pi = 0$) by Γ and its edge ($k_1 a / \pi = 1$) by X .

B. Experimental setup

Our experimental setup, though simple, is designed in such a general manner to facilitate modification at will of the EBCs applied at each electrode by connecting external circuits (resistive, capacitive, and/or inductive). The current version has substantially improved, as compared to the one used in Ref. 26. First, the plate is inserted into a 1 mm-pitch PCI Express[®] edge card reader (stackable female connector with 98 contacts), allowing connection of the electrodes of the crystal to different electric impedances, through controlled switches (FST3125). All connections are now realized in a secure and robust manner avoiding any noise and parasitic effects arising from soldering. Second, a microcontroller (Teensy 3.6) is used to actuate these switches, thus making possible a real-time control. Apart from the first pair of electrodes reserved for the excitation of the plate, the remaining 47 pairs of electrodes will be used to load the plate with external circuits, as shown in Fig. 2(a). Only this part of the plate (we will refer to it as loaded crystal hereafter) will be utilized to image the frequency response of the structure, except otherwise stated.

A 200V-amplitude and 0.1 μs -width pulse signal is applied to the first pair of electrodes at the one edge of the piezoelectric plate, using a Panametrics 5058PR pulse generator, to excite guided waves within the plate. The incident beam penetrates into the loaded crystal and is reflected back at the other edge. To visualize the frequency response of the crystal, we employ an all-electric experimental technique (more details can be found in Ref. 26). After excitation of the piezoelectric plate, any deformations producing not negligible thickness variations of the plate will be manifested as electric potential variations on the electrodes through the electromechanical coupling that takes place within the piezoelectric material; we exclusively measure these electric potentials along the plate. To this end, a second PCI card reader, identical to the first one used for the loading by external circuits, is connected to the remaining free edge of the plate [Fig. 2(b)]. The electric potential is recorded with a 10-bit quantification, at each electrode position, on the upper or lower side of the plate, with the help of a digital oscilloscope (LeCroy HRO66ZI WaveRunner), the ground of the pulse generator being taken as reference;

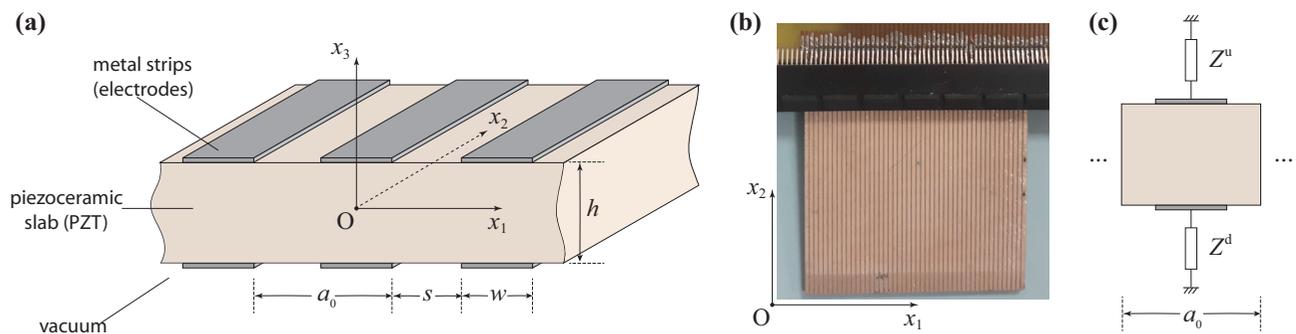


FIG. 1. (a) Schematic representation of the one-dimensional piezoelectric crystal, extended to infinity along x_1 - and x_2 -directions coinciding with the transversely isotropic plane of the piezoceramic material, poled across its thickness along the x_3 symmetry axis. (b) An image of the fabricated sample with finite dimensions ($48 \text{ mm} \times 48 \text{ mm} \times 1.6 \text{ mm}$), used in the experiments; the metallic strips (width $w = 0.7 \text{ mm}$ and separation gap $s = 0.3 \text{ mm}$) aligned along x_2 -direction are connected at the top edge of the plate to a PCI reader. The structure consists of $N = 48$ elementary blocks of length $a_0 = 1 \text{ mm}$. In (c) an elementary block of the structure loaded with electric circuits of impedance Z^u (upper plate side) and Z^d (lower plate side).

TABLE I. Material parameters for PZT (Navy VI), used in the calculations.

Material Property	Symbol	Value ^a
Elastic coefficients c_{pq}^E [GPa]	c_{11}^E	114.0
	c_{12}^E	75.7
	c_{13}^E	70.0 (72.4)
	c_{33}^E	107.0 (111.0)
	c_{44}^E	20.0 (26.3)
	c_{66}^E ^b	19.2
	Piezoelectric coefficients e_{ip} [C m^{-2}]	e_{15}
	e_{31}	-2.92
	e_{33}	20.3 (23.4)
Relative permittivity coefficients ϵ_{pq}^S	ϵ_{11}^S	2120.
	ϵ_{33}^S	1980.
Mass density [kg m^{-3}]	ρ	7510. ^c (7780.)

^a In the parenthesis, we give —if different— the initial values as provided by the manufacturer²⁸.

^b $c_{66}^E = \frac{1}{2}(c_{11}^E - c_{12}^E)$.

^c Measured.

the time window chosen is typically set to $250 \mu\text{s}$, with a sampling period equal to 5 ns , allowing to observe a few forward and backward traveling waves after reflections at the edges of the plate. Each measured signal is then averaged 30 times to improve the signal-to-noise ratio. The above settings ensure a sufficiently high Nyquist frequency and accurate resolution of the spectra obtained through fast Fourier transforms (FFT).

191 C. Simulations and signal processing

192 All band structure calculations shown in this paper
193 were obtained by the use of a finite-element commercial
194 package²⁷ (except otherwise stated). Assuming trans-
195 lational invariance along x_2 -direction, the unit cell is

196 reduced to a rectangle and the metallic strips to lines
197 of zero thickness, able to host surface electric charges.
198 The classical constitutive equations of piezoelectricity
199 (stresses as functions of strains and electric field) apply
200 at the surface of the rectangle. The boundary condi-
201 tions are: (i) continuity of all fields between adjacent
202 cells and periodic (Bloch) conditions at the left and right
203 boundaries of the same cell, and (ii) at the upper and
204 lower edges of the cell, free stress conditions, together
205 with discontinuity of the normal component of the elec-
206 tric displacement field related to the surface charge den-
207 sity across the interface, assuming a uniform distribu-
208 tion of charge along the line representing the metallic strip.
209 Under this last assumption, the whole electrode is seen
210 as an electric node, where Ohm's relation involving ex-
211 ternal impedance loads can be applied. We recall that
212 a full comparison between an ab-initio analytical model
213 using the above assumptions and this FEM approach was
214 given in Ref. 25, leading to practically identical (graphi-
215 cally indiscernible) results for the band diagrams of these
216 systems. We note in passing that only the positive part of
217 the BZ will be represented, since the dispersion diagrams
218 are symmetric with respect to the vertical (frequency)
219 axis, in all cases discussed here. The material param-
220 eters (elastic, piezoelectric and coupling constants) for
221 PZT used in the simulations are shown in Table I and
222 were determined by adjusting the dispersion relation of
223 a floating-potential crystal to the corresponding experi-
224 mental data. The experimental dispersion relation $\omega(k_1)$,
225 $\omega = 2\pi f$ being the angular frequency, was obtained, af-
226 ter applying a 2D FFT of the potential signals $V(x_{1,n}, t)$
227 measured at each electrode position $x_{1,n}$, $n = 2, 3, \dots, 48$,
228 of the upper and lower side of the corresponding finite
229 plate, following the procedure described in Ref. 26. The
230 simulation results agree very well with the experimental
231 data (tolerance better than 3.2%).

232 The floating-potential EBCs were used as a bench-
233 mark; we are, however, interested in more complex unit
234 cells. For the systems under study, the dispersion plot

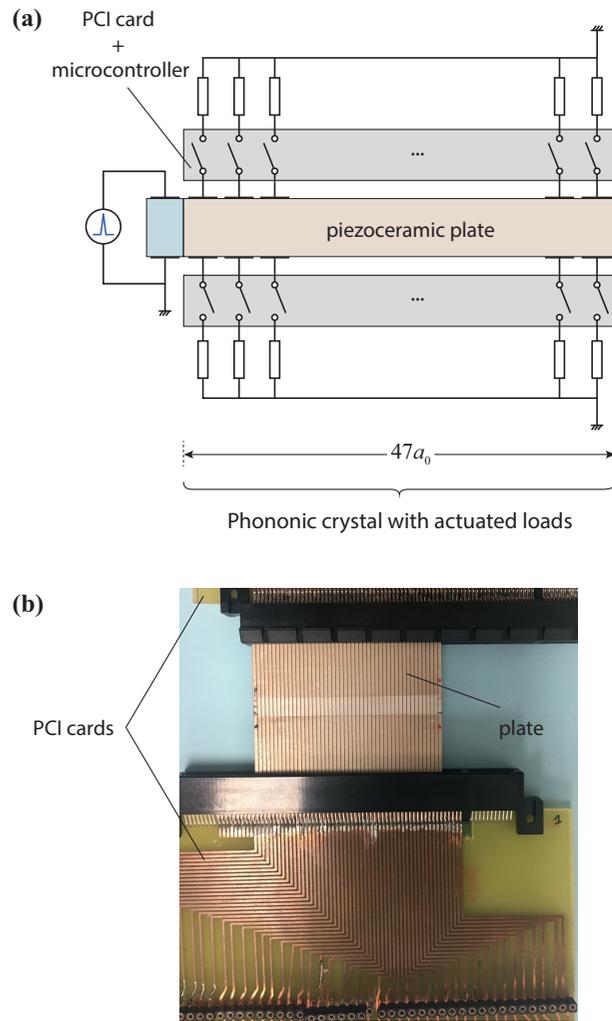


FIG. 2. The experimental setup used for the study of the phononic plate of Fig. 1: (a) schematic representation, and, (b) a view of the device. Excitation takes place at the left end of the plate and loading by external circuits at its remaining part through controlled EBCs.

235 is more crowded: several eigenmode trajectories coexist
 236 in the ω - k_1 space at neighboring points and cannot be
 237 easily resolved, after a 2D FFT is performed. To better
 238 resolve these separate contributions, we opt for a different
 239 method of analysis, by adapting an autoregressive model
 240 to two dimensions (AR2D), i.e., to both space and time
 241 directions²⁹. The results obtained in that manner agree
 242 with the corresponding picture obtained by a 2D FFT,
 243 without suffering from a wide ω - k_1 peak-character.

244 III. RESULTS AND DISCUSSION

245
 246 In what follows, we will focus on three configurations,
 247 all of them using inductance loads on the unit cell of the
 248 crystal, shown schematically in the left panel of Fig. 3;

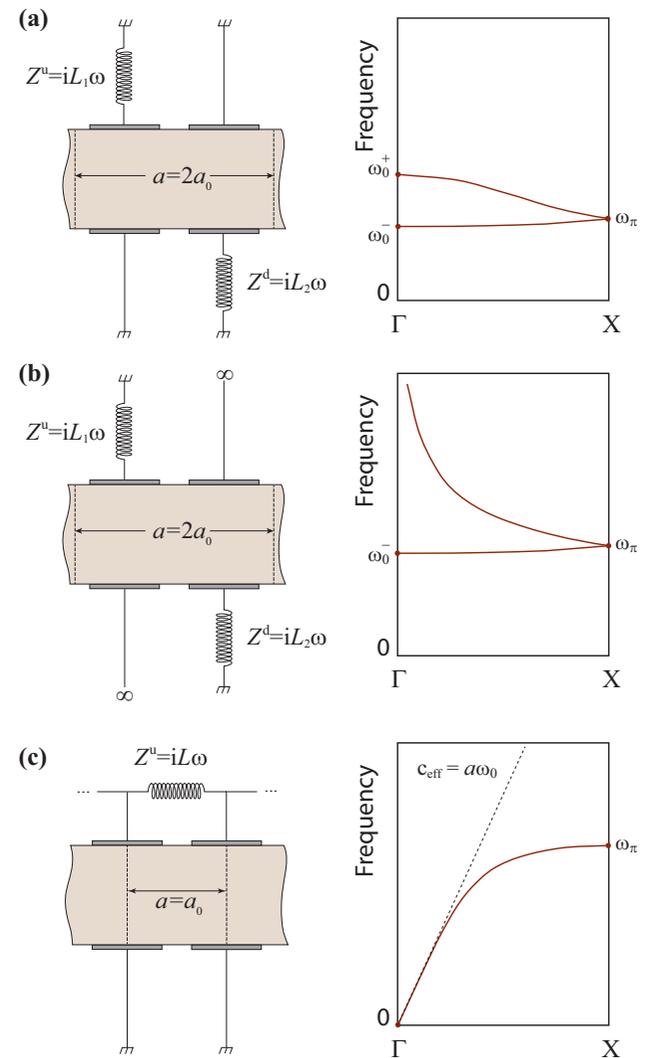


FIG. 3. The three configurations with inductive loads under study. Left panel: the unit cell of each crystal with different electric boundary conditions applied at the metallic strips [(a) grounded-inductance two-atom crystal, (b) floating-potential-inductance two-atom crystal, and (c) intercellular adjacent inductance-loaded crystal]. Right panel: schematic representation of the corresponding dispersion relation of the induced LC-origin bands (for $L_1 = L_2$, when applicable).

249 their interest lies on the unusual multidispersive form
 250 of localized and/or extended frequency bands of inductive
 251 origin that this circuitry may offer. More precisely,
 252 the first two cases correspond to two-atom unit cells
 253 ($a = 2a_0$), constructed by combination of two antiparallel
 254 elementary blocks (of width a_0), each of them having
 255 a L -load on the one side of the plate, the other-side
 256 electrode being either shunted [Fig. 3(a)] or in floating
 257 potential [Fig. 3(b)]. Though inspired from Ref. 26, the
 258 resulting structures offer new possibilities, since they possess
 259 inversion symmetry and exhibit, respectively, a set
 260 of two flat bands of localized modes and a set of one
 261 flat and one infinitely extended hyperbola-like frequency

bands, as schematically depicted in the right panel of Fig. 3. A third configuration [see Fig. 3(c)] consists of a L -load connecting adjacent strips, the EBCs conserve, in that case, the spatial periodicity of the metallic array ($a = a_0$). The corresponding modes originating from this circuitry exhibit a linear dispersion behavior with an effective medium slope $c_{\text{eff}} = a\omega_0$ at the long-wavelength limit which bends and tends to a zero-group-velocity branch at X point. It is worth noting that such a configuration is a direct analogue of the unit cell of a transmission line governed by the well-known telegrapher's equation.

The different characteristic angular frequencies ω_0^\pm , ω_0 , ω_π , appearing in Fig. 3 to describe qualitatively the dispersion of these eigenmodes at Γ and X points of the BZ, depend of course of the choice of L and of some characteristic capacitance values C , C_s , C_1 , and C_2 that describe effectively the intrinsic behavior of the piezoelectric plate in terms of equivalent circuits. More details are given in Appendix A. We note that this analysis for the two-atom crystals reveals higher-order interactions of capacitive origin within the plate which takes place between neighbors of adjacent cells and opposite sides, compared to previous demonstrations²⁶.

A. Grounded and inductance loaded two-atom unit cell

To begin with we shall examine the case of up- and down-side inductance loads alternated with grounded strips. The unit cell is thus a two-atom molecule analog with inversion symmetry if $L_1 = L_2$, or with identity operation symmetry only, if $L_1 \neq L_2$, as shown in Fig. 3(a). Aiming to analyze its frequency band structure both theoretically and experimentally, we find it useful to focus first on the frequency response of the corresponding monoatomic crystal of lattice constant $a = a_0$ [suppose only the left half part of the unit cell of Fig. 3(a)]. In the absence of piezoelectric coupling ($e_{ij} = 0$), the eigenmodes of the crystal are separated into two independent, non-interacting subspaces: the elastic-displacement Lamb guided modes of the plate (labelled, hereafter, with A for antisymmetric and S for symmetric modes, following standard notation used for Lamb modes in plates), simply folded at the BZ edge ($k_1 a_0/\pi = 1$) and the EM modes which originate from an LC -equivalent circuit behavior of the unit cell. The electric resonances of these individual LC atoms interact between each other and form a cos-like resonant band. In Fig. 4(a) we show the calculated —using a finite-element method— frequency band structure of such a crystal, the first subspace shown in gray lines, the second one in blue dotted line, extending from 1.294 MHz ($k_1 = 0$) to 0.597 MHz ($k_1 = \pi/a_0$). We note in passing that this zero coupling analysis, already presented in previous works^{24,26}, can be employed to visualize the bands of EM origin before hybridization.

When two antiparallel atoms are brought together to form a two-atom unit cell as shown in Fig. 3(a) the BZ

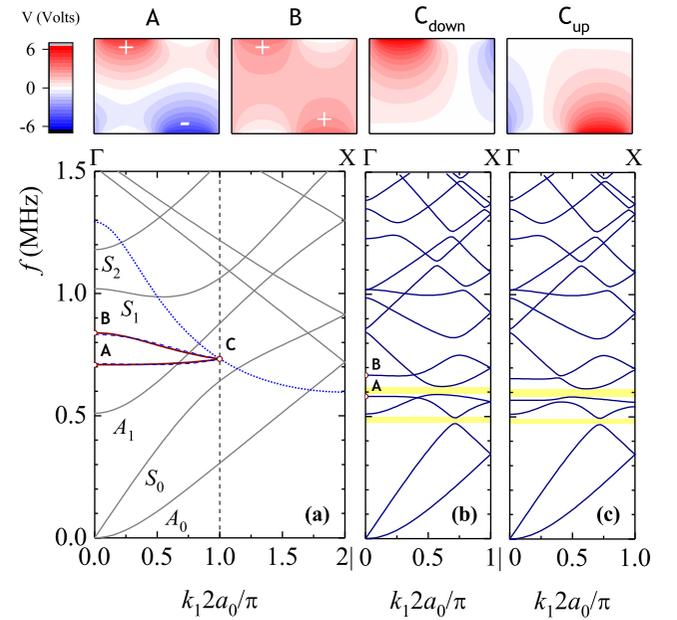


FIG. 4. (a) Calculated frequency band structure for a monoatomic crystal of lattice constant $a = a_0$ [only the left half part of the unit cell in Fig. 3(a) with $L_1 = 30 \mu\text{H}$ is considered] when piezoelectric coupling is switched off. The blue dotted curve spanning the whole BZ indicates the EM resonance band, while the thick red line represents the same band for the diatomic crystal of lattice constant $a = 2a_0$ [unit cell of Fig. 3(a) with $L_1 = L_2 = 30 \mu\text{H}$]; the gray lines correspond to Lamb eigenmodes of the phononic plate, labelled following standard notation. The full calculation of the band structures for the diatomic crystal of Fig. 3(a) when piezoelectric coupling is taken into account is shown in (b) for $L_1 = L_2 = 30 \mu\text{H}$ and (c) for $L_1 = 30 \mu\text{H}$ and $L_2 = 33 \mu\text{H}$. Yellow-shaded regions denote absolute frequency gaps. Top panel: Potential field plots within a unit cell for the points labelled in (a).

becomes shorter ($a = 2a_0$). An additional folding at $k_1 a_0/\pi = 0.5$ of the dispersion plot is dictated and, as a result, the EM resonant band is thus split into two separate flat minibands, a more dispersive, $\omega^+(k_1)$, extending from 0.839 MHz ($k_1 = 0$) to 0.732 MHz ($k_1 = \pi/a$), and a less dispersive, $\omega^-(k_1)$, extending from 0.708 MHz ($k_1 = 0$) to 0.732 MHz ($k_1 = \pi/a$). These bands compare very well (relative error better than $\pm 0.9\%$) to a tight-binding description³⁰ with first-neighbour interactions between atomic sites [blue dashed lines in Fig. 4(a)] that leads to $\omega^\pm(k_1) = [\omega_{\text{at}}^2 + 2\gamma_1 \cos k_1 a \pm 2\gamma_2 \cos(k_1 a/2)]^{1/2}$, $\gamma_1 = \gamma(a)$, $\gamma_2 = \gamma(R)$ being the overlap integrals γ depending on the distance $a = 2a_0$ and $R = \sqrt{a_0^2 + h^2} = 1.89a_0$ between the centers of the atomic potential functions and ω_{at} the eigenfrequency of an isolated atom. By a best fitting procedure we find $\omega_{\text{at}}/(2\pi) = 0.753$ MHz, $\sqrt{\gamma_1}/(2\pi) = 0.128$ MHz, and $\sqrt{\gamma_2}/(2\pi) = 0.214$ MHz. The $\omega^\pm(k_1)$ bands are independently reproduced by the electric-circuit picture [see Eq. (A1) in Appendix A1, with $C = 0.103$ nF, $C_s = 0.552$ nF, $C_1 = 0.0348$ nF, and

337 $C_2 = 0.116$ nF obtained by a best fitting procedure], also
 338 in accordance with the above tight-binding model.

339 A more careful look at the potential field functions at Γ
 340 point plotted within the unit cell (see top panel of Fig. 4)
 341 reveals a bonding ($\Phi_+ = C_+(\phi_1 + \phi_2)$) and antibonding
 342 ($\Phi_- = C_-(\phi_1 - \phi_2)$) character in analogy with a diatomic
 343 molecule picture in quantum physics, ϕ_1 and ϕ_2 being the
 344 two atomic potential functions and C_{\pm} appropriate coeffi-
 345 cients. Along ΓX direction the potential fields evolve
 346 progressively towards a more weighted one-atom picture
 347 with a double-degenerate state at X point (see two right
 348 plots of the top panel in Fig. 4), this degeneracy being
 349 lifted when inversion symmetry disappears ($L_1 \neq L_2$).

350 This picture is further modified when piezoelectric cou-
 351 pling is taken into account ($e_{ij} \neq 0$), as it should be in
 352 a real system. The result of our calculation is shown in
 353 Fig. 4(b). As a first remark, one observes a blue shift
 354 for the Lamb guided modes and a red shift for the EM
 355 origin resonant bands. And more importantly, the two
 356 eigenmode subspaces interact between each other and, in
 357 general, one expects the formation of avoided crossings
 358 between modes of the same symmetry. The lower in fre-
 359 quency EM resonant band $\omega^-(k_1)$ interacts with both the
 360 S_0 branch and the first folded A_0 branch, and similarly
 361 the higher in frequency EM band $\omega^+(k_1)$ interacts with
 362 both the A_1 branch and the first folded S_0 branch, thus
 363 leading to narrow selective absolute gaps (yellow-shaded
 364 regions). A slight detuning between L_1 and L_2 lowers
 365 further the symmetry of the system (inversion is now in-
 366 valid) thus resulting in interactions, though very weak,
 367 between all bands [see Fig. 4(c)].

368 Next, we confirm the above predictions, by their experi-
 369 mental verification. We measured the potential signal
 370 $V(x_{1,n}, t)$ along the $n = 2, 3, \dots, 48$ electrode positions
 371 located at the upper side of the plate, and after appli-
 372 cation of the AR2D model, we obtain the experimental
 373 $V(\omega, k_1)$, whose amplitude is shown in Fig. 5(a), and
 374 close to it, in Fig. 5(b), the calculated frequency band
 375 structure shown in Fig. 4(b). The different branches are
 376 color-indexed with the value of the potential, averaged on
 377 the area of the upper-side electrodes: the darker the tra-
 378 jectory, the weaker the electromechanical coupling, that
 379 takes place within the plate, is.

380 The experimental dispersion plot mainly reveals the
 381 S_0 -like branches of positive slope in the positive part
 382 of the 1st and negative part of the 2nd BZ, that com-
 383 pare well to the corresponding theoretical curve with effec-
 384 tive medium slope of 3013 m s⁻¹ at the long wave-
 385 length limit. In addition, one observes some bright spots
 386 spanning the frequency region between 0.6 and 0.7 MHz,
 387 that could correspond to the localized modes of EM
 388 origin in Fig. 5(b) at about 0.58 MHz for $\omega^-(k_1)$ and
 389 at about 0.67 MHz for $\omega^+(k_1)$. At higher frequencies
 390 but below 1 MHz, two more trajectories are experi-
 391 mentally observed, belonging probably to A_1 -like and S_0 -like
 392 branches, visible in the positive part of the 1st and 2nd
 393 BZ, respectively (but also in the negative part of the 2nd
 394 and 1st BZ). The comparison between theory and experi-

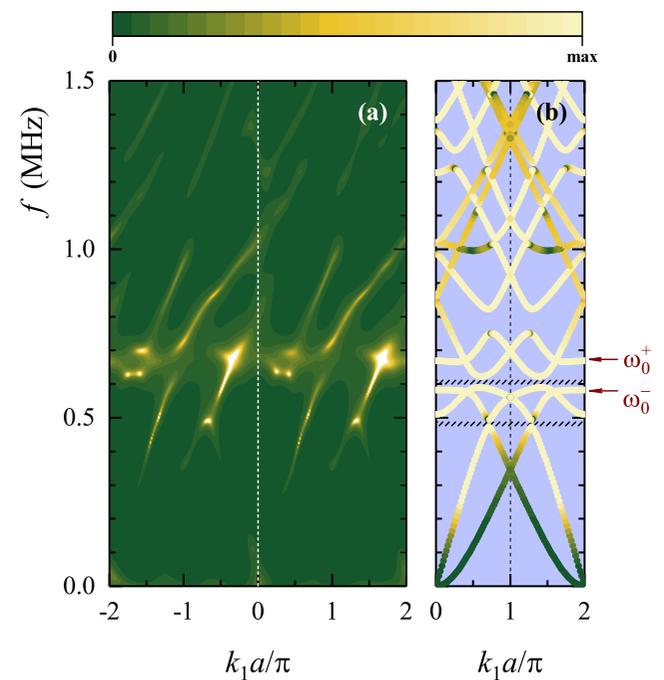


FIG. 5. Frequency band structure of the diatomic grounded- L piezoelectric phononic crystal (lattice constant $a = 2a_0 = 2$ mm) with EBCs depicted in Fig. 3(a). (a) Experimental dispersion plot for the finite crystal, consisting of $N = 24$ unit cells, obtained by use of the AR2D model on the electric-potential signal recorded at the upper-side electrode positions. (b) Calculated dispersion plot obtained through finite-element numerical simulations for the corresponding infinite crystal for $L = 30$ μ H. In (a) the color map is saturated; in (b) the trajectories are color-indexed with the average potential on the upper-side electrodes of the unit cell. Red arrows and hatched areas denote, respectively, frequency bounds of the EM bands at Γ point and absolute gaps.

395 ment is, however, not straightforward, especially because
 396 the finite-element simulations predict much more active
 397 branches than seen experimentally.

398 B. Floating potential and inductance loaded two-atom 399 unit cell

400 If we replace the two grounded ends of the previous
 401 structure by free floating potential (FP) ends, we obtain
 402 the unit cell shown in Fig. 3(b). A similar methodol-
 403 ogy can be developed as that given in Sec. III A in order
 404 to follow the formation of the resonant bands; the main
 405 difference lies in the specific form of this band before hy-
 406 bridization. As it can be easily shown³¹ this branch is
 407 hyperbolic for the corresponding monatomic crystal [con-
 408 sider only the left half part of the unit cell of Fig. 3(b)]
 409 and when constructing the diatomic unit cell, the fold-
 410 ing results in two branches, one hyperbola-like, $\omega^+(k_1)$,
 411 extended in frequency, the other, $\omega^-(k_1)$, relatively flat.
 412 When $L_1 = L_2$ these two distinct bands degenerate to

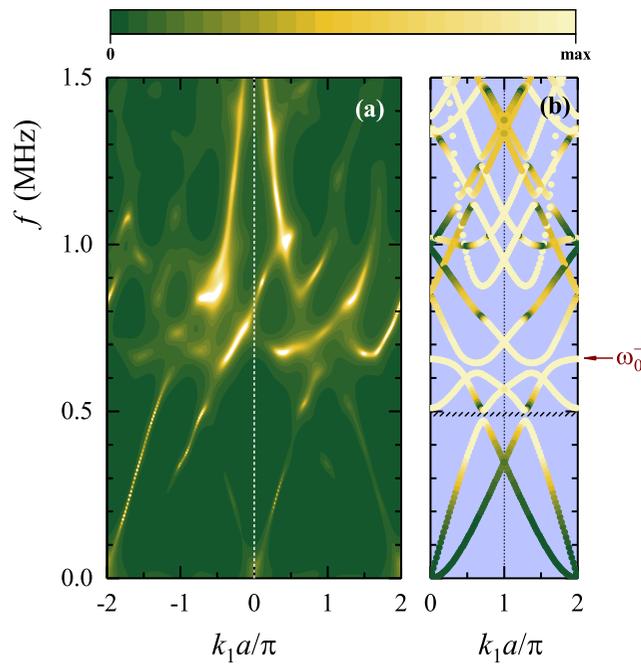


FIG. 6. Frequency band structure of the floating-potential- L piezoelectric phononic crystal (lattice constant $a = 2a_0 = 2$ mm) with EBCs depicted in Fig. 3(b). (a) Experimental dispersion plot for the finite crystal, consisting of $N = 24$ unit cells, obtained by use of the AR2D model on the electric-potential signal recorded at the upper-side electrode positions. (b) Calculated dispersion plot obtained through finite-element numerical simulations for the corresponding infinite crystal for $L = 30 \mu\text{H}$. In (a) the color map is saturated; in (b) the trajectories are color-indexed with the average potential on the upper-side electrodes of the unit cell. Red arrow and hatched area denote, respectively, frequency bounds of the EM band at Γ point and absolute gap.

413 a common frequency ω_π at X point; again this degeneracy
414 originating from the inversion symmetry of the unit
415 cell, is lifted if $L_1 \neq L_2$. The finite frequency point
416 $\omega_0^- = \omega^-(k_1 = 0)$ can be tuned through the choice of L .
417 Analytic expressions are derived in terms of an equivalent
418 periodic transmission-line picture, as detailed in Ap-
419 pendix A 2.

420 The experimental frequency band structure of our
421 finite-size sample is obtained following the same proce-
422 dure, as for the previous case of grounded crystal
423 (Sec. III A). The results are shown in Fig. 6(a), and next
424 to it [Fig. 6(b)] the calculated frequency band structure
425 for the corresponding infinite crystal, assuming again a
426 pure L -component ($L = 30 \mu\text{H}$). In the experimental
427 picture, three frequency regions are distinguished: be-
428 low 0.6 MHz the S_0 -like and A_0 -like branches are clearly
429 identified, in good agreement with those predicted theo-
430 retically, and this was also the case for the grounded
431 crystal (Fig. 5). Next, for frequencies above 0.6 MHz
432 and below 1.0 MHz some bright spots are observed but a
433 one-to-one comparison to the theoretical picture becomes

434 not straightforward. Finally, above 1.0 MHz the two ex-
435 tended, hyperbolic branches become dominant and are
436 clearly observed within the 1st BZ, in both its positive
437 and negative part.

438 In both cases examined up to now, we remark that the
439 experimental dispersion plots [Figs. 5(a) and 6(a)] do not
440 reveal all branches predicted theoretically, and, more im-
441 portantly, symmetry operations such as translation sym-
442 metry by a reciprocal lattice vector, $\omega(k_1) = \omega(k_1 + \frac{2\pi}{a})$,
443 and/or mirror symmetry, $\omega(k_1) = \omega(-k_1)$, are not ob-
444 served for all trajectories. This could be explained by
445 finite size effects and excitation that favors some of the
446 branches with respect to others. To facilitate compari-
447 son to the theoretical picture, we artificially restore the
448 above said lost symmetries, by taking the average on the
449 positive- k_1 and negative- k_1 part after, i) horizontal flip-
450 ping, and, ii) horizontal translation by 2 (in reduced k_1 -
451 axis units) of the latter. These two operations lead to
452 a picture with mirror symmetry with respect to vertical
453 (frequency) axis at both Γ and X points, thus the ΓX
454 direction is sufficient for any analysis. The results are
455 shown in Fig. 7, for the grounded crystal [left panel, plot
456 (a)] and for the floating-potential crystal [right panel,
457 plot (c)]. Their comparison to the corresponding theo-
458 retical predictions [Figs. 5(b) and 6(b)] reveals a red shift
459 of the simulated EM modes. Obviously, the reality, far
460 away from the numerous simplifications assumed in the
461 simulations, includes finite size effects, different elastic
462 boundary conditions at the edges of the plate due to the
463 use of the PCI cards, and, real-electronics effects, present
464 in the circuitry, that were neglected in the previous analy-
465 sis. In fact, a Z_e impedance load (see Appendix B) will be
466 considered instead of the idealized $Z^u = Z^d = Z_L = iL\omega$
467 generated by a pure L -component. In plots (b) and (d)
468 of Fig. 7 we show the calculated band structures that
469 correspond to those of Figs. 5(b) and 6(b), but for the
470 real load Z_e [Eq. (B1)] instead of the pure inductive load
471 with $L = 30 \mu\text{H}$. The use of Z_e blue-shifts the EM
472 modes [by about $\sim 10\%$ for ω_0^\pm with respect to those
473 of Figs. 5(b) and 6(b), indicated here by red arrows as
474 guides to the eye] and improves the comparison between
475 experimental data and theoretical simulations. The EM
476 modes manifest themselves as two flat bands lying in the
477 frequency range from 0.6 to 0.75 MHz for the grounded-
478 L crystal [Fig. 7(b)] in rather good agreement with the
479 experimental picture [bright spots extending from 0.63
480 to 0.71 MHz in Fig. 7(a)]. For the floating-potential- L
481 crystal [Fig. 7(d)] the EM modes manifest themselves, on
482 the one hand, as a flat band centered at 0.7 MHz and, on
483 the other hand, as a hyperbolic branch which interacts
484 strongly with the A_1 -like branch and spans the whole
485 frequency range above 0.9 MHz; these compare well to
486 the experimental picture [Fig. 7(c)] that reveals, respec-
487 tively, a flat bright segment from 0.66 to 0.75 MHz and
488 a hyperbolic branch above 0.82 MHz.

489 We note in passing the appearance of narrow, selective,
490 absolute frequency gaps for both systems [see hatched ar-
491 eas in Figs. 7(b), (d)] originating from avoided-crossing

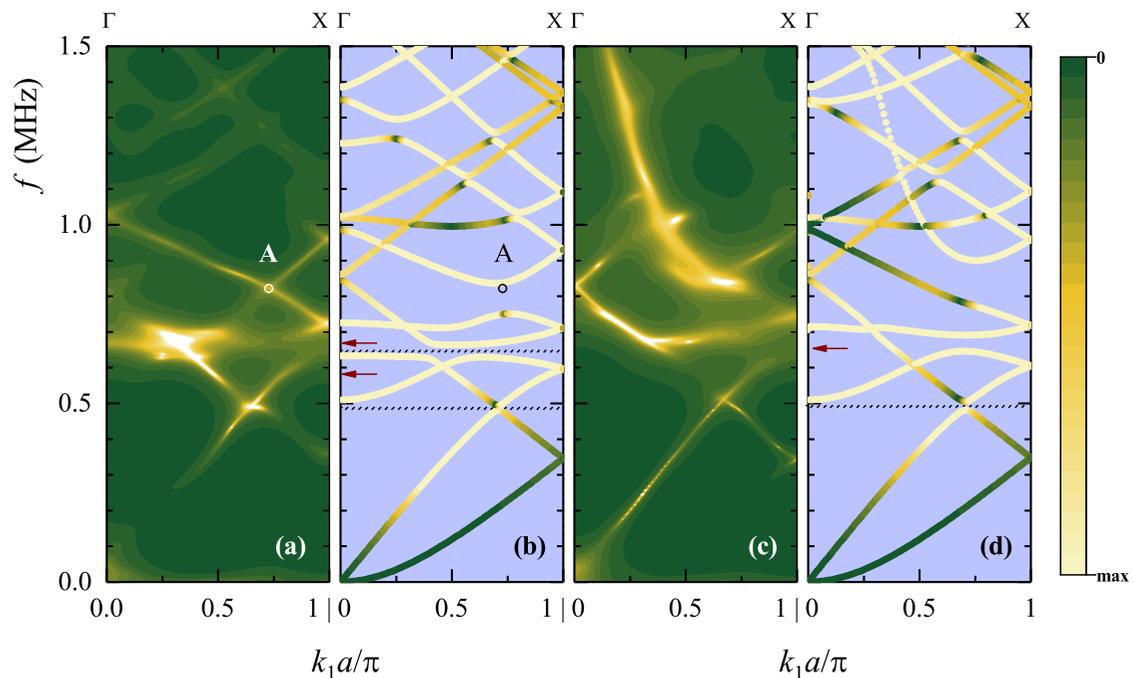


FIG. 7. Experimental frequency band structure of the diatomic (a) grounded- L and (c) floating-potential- L piezoelectric phononic crystals, after reduction along ΓX direction (see text) of their initial map plots, shown respectively in Figs. 5(a) and 6(a). In (b) and (d) the corresponding calculated dispersion plots for the infinite crystals, assuming real loads Z_e instead of the pure inductive ones Z_L ($L = 30 \mu\text{H}$). Red arrows denote the frequency positions of the EM bands at Γ point as predicted for the case of Z_L instead of Z_e [see Figs. 5(b) and 6(b), respectively]; hatched areas indicate, as usually, absolute gaps.

492 between bands of the same symmetry. Therefore, one ex-
 493 pects that even structures with a small number of units
 494 will exhibit such type of gaps³². Their frequency po-
 495 sition is indirectly controlled through ω_0 values scaling
 496 as $\sim L^{-1/2}$ [see Appendix A]. Their width is narrow,
 497 but possible enlargement could be achieved with elec-
 498 tronic components facilitating the degree of interaction
 499 [e.g., negative capacitors²⁵].

500 A careful global evaluation of the picture obtained by
 501 both models used for the simulations, i.e., Z_L and Z_e ,
 502 for the two crystals under study, suggests that none of
 503 these can perfectly describe the experimental picture in
 504 terms of frequency position of the EM modes (they are in
 505 general slightly shifted, but the hyperbolic branch clearly
 506 blue-shifted in the simulations) and in terms of degree of
 507 interaction that takes place in the avoided-crossing re-
 508 gions when EM and Lamb-like modes cross each other
 509 [weak avoiding is experimentally observed, while a strong
 510 one is predicted, e.g. at point A in Figs. 7(a), (b)].
 511 Deviations are also observed between some dark branch
 512 segments (vanishing potential value along them), while
 513 predicted highlighted in theory, e.g., the A_1 -like branch
 514 which is theoretically predicted with a cut-off frequency
 515 close to 0.5 MHz is not experimentally discernible. We
 516 deduce that even if a pure L -component is too simple
 517 to reproduce exactly the experimental picture of the dis-
 518 persion plot of these crystals, the use of the experimen-
 519 tally identified load Z_e is not sufficient either. A possible

520 reason could be the modification of this individual-atom
 521 load Z_e (we recall that its identification has been real-
 522 ized under isolated conditions, outside the crystal) when
 523 it is introduced in our device including the plate itself
 524 and the system of two PCI cards. On the other hand,
 525 simulations still constitute idealized models, since they
 526 neglect any finite size effects and leakage of elastic and
 527 EM modes in the surrounding medium (air). For the
 528 above reasons, comparison between theory and exper-
 529 iments remains, however, very satisfactory and provides
 530 a good verification of the main features of the dispersion
 531 properties of these crystals.

532 C. Intercellular inductance loaded unit cell

533 As a last case, we examine a crystal plate with inter-
 534 cellular connection of adjacent strips located at the same
 535 side of the plate (say the upper one) via an inductance
 536 load L , the other-side strips being grounded, as depicted
 537 in Fig. 3(c). These EBCs conserve the periodicity of the
 538 metallic array. The numerically predicted band struc-
 539 ture of that crystal (lattice constant $a = a_0$) is shown in
 540 Fig. 8 for two distinct inductance values: $L = 150 \mu\text{H}$
 541 [plot (a)] and $L = 30 \mu\text{H}$ [plot (b)]. The most striking
 542 feature of these dispersion plots is the appearance of a
 543 linear-dispersion branch at the long-wavelength limit, in
 544 addition to the traditionally expected S_0 -like and A_0 -like

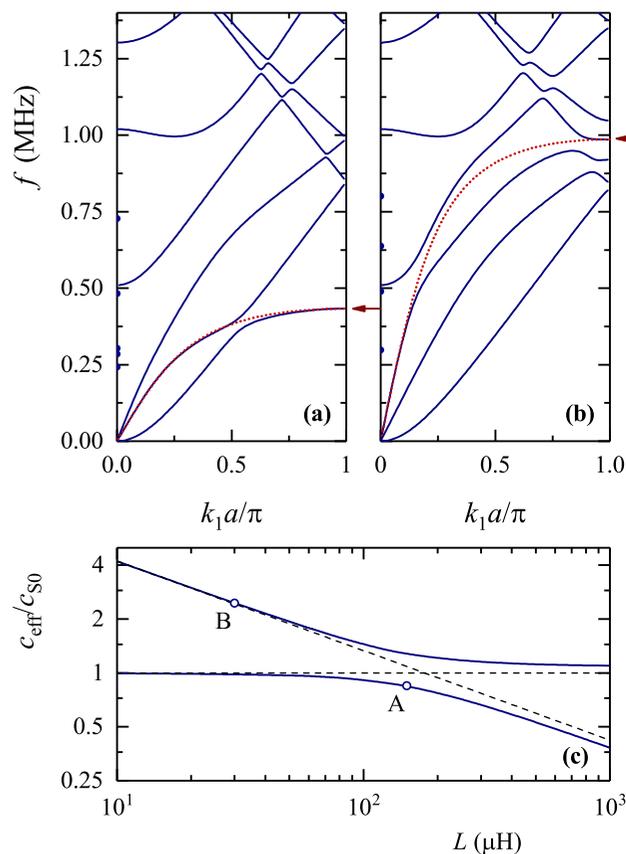


FIG. 8. Calculated frequency band structure for a monoatomic crystal of lattice constant $a = a_0$ whose unit cell is depicted in Fig. 3(c) with (a) $L = 150 \mu\text{H}$ and (b) $L = 30 \mu\text{H}$. The dotted red lines represent an estimate of the unhybridized L -induced band, applying Eq. (A3) for appropriate values of the parameters C , C_s (see text), with starting parameters $\omega_\pi / (2\pi)$ (indicated by red arrows) and the long-wavelength slope, c_{eff} , of this frequency branch. (c) Evolution of the slope of the two linear branches at $\omega \rightarrow 0$ for several values of the inductance load L . Horizontal and oblique dashed lines represent, respectively, the effective medium slope of the S_0 branch of the corresponding grounded crystal and the L -induced branch from Eq. (A3) calculated for C , C_s determined for $L = 10 \mu\text{H}$. Points A and B correspond to the L -induced bands (red dotted curves) of plots (a) and (b), respectively.

branches emerging at the same frequency range. The general form of that L -induced branch when the piezoelectric coupling is switched off ($e_{ij} = 0$) follows the schematic representation given in the right plot of Fig. 3(c) and is described by the relation (A3) for some appropriate values C , C_s in an equivalent periodic transmission line picture. The slope, c_{eff} , of this L -branch is tuned via the choice of L , and covers a wide range of effective medium velocities not easily encountered in ordinary materials. The equivalent transmission line model presented in Appendix A predicts a scaling $\sim L^{-1/2}$ for given (L -independent) C and C_s values (we find $C = 0.645 \text{ nF}$ and $C_s = 0.593 \text{ nF}$, for the crystal under study). When this

coupling is switched on ($e_{ij} \neq 0$), as it should be in a real system, the L -induced band interacts with the same symmetry bands of Lamb eigenmodes and generates avoided-crossing effects. To facilitate its visualization before hybridization, we apply the same relation [Eq. (A3), with different values of C , C_s , reflecting all coupling effects] to model its unhybridized form, represented in Fig. 8(a) and (b) by red dotted lines: after a careful reading of these dispersion plots, we identify the two starting parameters, ω_π (indicated by a red arrow), and, c_{eff} , the effective-medium slope of this branch at the long-wavelength limit, used to deduce the internal model capacitance parameters ($C = 0.737 \text{ nF}$ and $C_s = 0.868 \text{ nF}$, for $L = 30 \mu\text{H}$; $C = 1.413 \text{ nF}$ and $C_s = 0.900 \text{ nF}$, for $L = 150 \mu\text{H}$).

For the case of $L = 150 \mu\text{H}$ [Fig. 8(a)], the L -band with $c_{\text{eff}} = 2560 \text{ m s}^{-1}$ interacts weakly with the A_0 -like branch, and a small avoided-crossing occurs at about 0.375 MHz ; the other linear, S_0 -like, branch has $c_{\text{eff}} = 3850 \text{ m s}^{-1}$, which is higher than the corresponding value in a grounded crystal, $c_{S_0} = 3020 \text{ m s}^{-1}$. For the case of $L = 30 \mu\text{H}$ [Fig. 8(b)], the L -band with $c_{\text{eff}} = 7400 \text{ m s}^{-1}$ interacts weakly with the A_1 -like branch as well as with the first folded A_0 -like branch, giving rise to relatively larger avoided crossings, occurring, respectively, at about 0.50 MHz and 0.95 MHz ; the other linear, S_0 -like, branch has $c_{\text{eff}} = 2970 \text{ m s}^{-1}$, which is lower than c_{S_0} . The above suggest a possible interaction with the S_0 -like branch when the two modes are close enough. In Fig. 8(c) we plot the evolution of the slope at the long-wavelength limit, for several values of the inductance covering a large range spanning over two orders of magnitude, of both linear branches of a typical dispersion plot as those shown in Fig. 8(a) and (b). One clearly observes a hybridization between two distinct modes: the S_0 -like mode, represented by the horizontal dashed line, and the L -induced mode, represented by the oblique dashed line that corresponds to the $\sim L^{-1/2}$ scaling rule calculated at $L = 10 \mu\text{H}$ (for $C / [1 + C / (4C_s)] = 0.618 \text{ nF}$), far away from the crossing point that occurs in the vicinity of $L = 150 \mu\text{H}$. For this value of L , a high-degree hybridization takes place, both linear-dispersion modes carry both the S_0 and L -induced character, the point A corresponding to the L -induced band. On the contrary, for $L = 30 \mu\text{H}$ point B, that corresponds to the L -induced band, conserves mainly its L -induced character.

We turn now our attention to the comparison with experimental results. As previously, the Z_e load will be considered instead of the idealized Z_L in the calculations. We measured the potential signal along the $n = 2, 3, \dots, 48$ electrode positions located at the upper side of the plate, and after application of the AR2D model, we obtain the experimental $V(\omega, k_1)$ whose amplitude is shown in Fig. 9(a) that compares well with the calculated frequency band structure including the real load Z_e , in Fig. 9(b).

The linear-slope, effective-medium branch of EM origin, clearly observed in the experiments, is perfectly re-

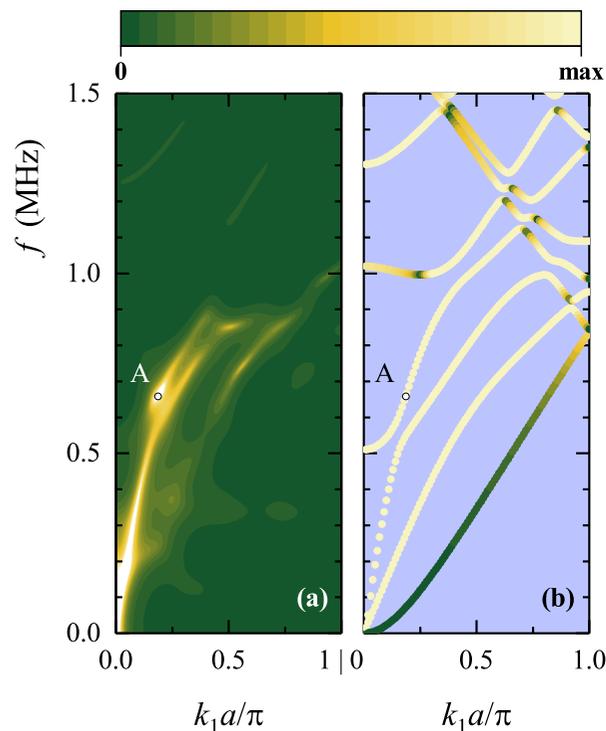


FIG. 9. (a) Experimental and (b) calculated frequency band structure for the monoatomic crystal of lattice constant $a = a_0$ whose unit cell is depicted in Fig. 3(c), loaded with impedance Z_e [Eq. (B1)]. In (b) the dispersion lines are color-indexed with the projection of the potential value averaged in the upper-side electrode of the unit cell.

616 produced in the calculations. The same holds for the
 617 bright part of the A_1 -like branch [see point A in Fig. 9(a)],
 618 though the latter is not discernible experimentally close
 619 to its cut-off frequency at ~ 0.5 MHz (this was also
 620 the case for the crystals presented in Sec. III A and
 621 III B). The S_0 -like branch is also observed experimen-
 622 tally, though with lower amplitude than predicted in the
 623 theoretical calculation. We note that the latter fails to
 624 capture the degree of repulsion of the trajectories in the
 625 vicinity of point A and for frequencies above it (this
 626 avoiding is predicted to be larger than experimentally
 627 observed). We close this part by a general remark con-
 628 cerning the A_0 -like branch which is not observed (apart
 629 some slightly visible parts close to X point), since the po-
 630 tential takes very low values along it, in all cases studied
 631 here, in perfect agreement with our numerical calcula-
 632 tions.

633 IV. CONCLUSIONS

634
 635 In conclusion, we have presented a thorough analy-
 636 sis of the dispersion properties of piezoelectric phononic
 637 plates, structured with metallic arrays on their surfaces

638 and loaded with inductive circuits. These structures,
 639 studied experimentally and theoretically, exhibit a va-
 640 riety of localized and/or extended in frequency modes
 641 that originate from an electric circuit behavior related to
 642 the inductive loads and span the low, intermediate, or
 643 high frequency range in a typical dispersion plot. This
 644 low-pass, band-pass, and high-pass behavior that mani-
 645 fests itself as an important potential variation along these
 646 trajectories, combined to the automatized, controllable
 647 character of our experimental device, constitute a power-
 648 ful tool for applications targeting real-time manipulation
 649 of elastic waves via EM waves and vice-versa. We demon-
 650 strate the appearance of unusual, high-valued, positive or
 651 negative group-velocity branches, not encountered in typ-
 652 ical Lamb-like dispersion plots of phononic plates, easily
 653 tuned via the external inductive loads.

654 Our experimental results, enhanced by non-ordinary,
 655 high-resolution signal processing techniques, are in ac-
 656 cordance with the theoretical predictions —despite some
 657 isolated discrepancies; they confirm the basic underlying
 658 mechanisms analyzed in this paper and the main phe-
 659 nomenological aspects that interest us for applications, our
 660 aim being among others the realization of viable, simple
 661 devices that can operate under real conditions and pro-
 662 duce the desired effects in the modulation of the disper-
 663 sion properties of these crystals. Nevertheless, although
 664 its very application-oriented aspect at a first sight, this
 665 study councils *per se* an important physical insight.

666 DATA AVAILABILITY STATEMENT

667 The data that support the findings of this study are
 668 available from the corresponding author upon reasonable
 669 request.

670 Appendix A: Equivalent electric circuits

671 A piezoceramic plate with metallized surfaces can be
 672 effectively described by an equivalent capacitor. When
 673 metallic strips are structured periodically on its both sur-
 674 faces, we can use an equivalent picture of a periodic trans-
 675 mission line whose unit cell coincides with the unit cell of
 676 the real structure. We give in what follows the electric-
 677 circuit models that correspond to such a description for
 678 the cases studied in this paper.

679 1. Two-atom grounded- L crystal

680 The equivalent circuit that models the two electric res-
 681 onators shown in Fig. 3(a) when the piezoelectric cou-
 682 pling is switched off is depicted in Fig. 10(a). We define
 683 all voltages in the input, V_{A_j} , and output, $V_{B_j} = \mathcal{P}V_{A_j}$,
 684 of the unit cell with respect to a common ground refer-
 685 ence, where $j = 1, 2$, and $\mathcal{P} = e^{-ik_1 a}$ is the Bloch phase
 686 factor (we assume an $e^{+i\omega t}$ time dependance in all fields);

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184

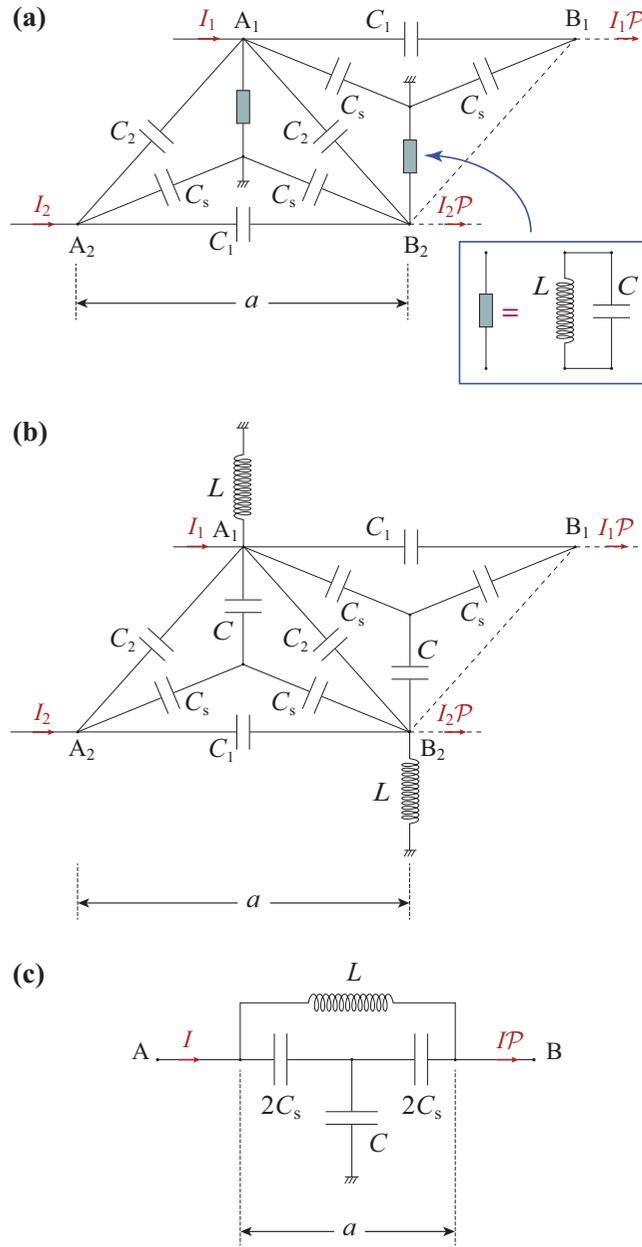


FIG. 10. The unit cell of a periodic transmission line describing equivalently (a) the two-atom grounded- L crystal [Fig. 3(a)], (b) the two-atom floating-potential- L crystal [Fig. 3(b)], and (c) the intercellular- L crystal [Fig. 3(c)], when electromechanical coupling is switched off.

687 the incoming and outgoing electric currents are respec-
688 tively I_j and $\mathcal{P}I_j$. Application of Kirchhoff's current and
689 voltage laws leads to a secular equation depending only
690 on \mathcal{P} and the set of unit-cell impedances. We thus obtain
691 the following dispersion equation for the two electric
692 resonant bands

$$\omega^\pm(k_1) = \left[LC \left(1 + 2 \frac{C_s + C_1 + C_2}{C} \right) \right. \quad (\text{A1})$$

$$\left. - 2 \frac{C_1}{C} \cos k_1 a \mp 2 \frac{C_2}{C} \cos \frac{k_1 a}{2} \right]^{-1/2}.$$

693 At Γ point (center of the BZ) the two distinct eigen-
694 frequencies are $\omega_0^+ = [LC(1 + 2\frac{C_s}{C})]^{-1/2}$ and $\omega_0^- =$
695 $[LC(1 + 2\frac{C_s + 2C_2}{C})]^{-1/2}$, corresponding to a *bonding* and
696 an *antibonding* mode; at X point (edges of the BZ) a
697 double degenerate eigenfrequency is found to be $\omega_\pi =$
698 $[LC(1 + 2\frac{C_s + 2C_1 + C_2}{C})]^{-1/2}$. The width $\Delta\omega^\pm = |\omega_0^\pm -$
699 $\omega_\pi|$ of these cos-like resonant bands is finite, and controlled
700 by L and the internal effective parameters of the model
701 C, C_s, C_1 and C_2 . We note that these capacitors
702 do not correspond to real electronic elements in our device,
703 but they describe its equivalent behavior. C and C_s
704 account for a capacitive effect between electrodes located
705 at the two opposite sides (i.e., along x_3 -direction) and at
706 the same side (i.e., along x_1 -direction) of the plate, respec-
707 tively. C_1 and C_2 describe the capacitive interaction
708 between electrodes of the same side at distance $a = 2a_0$
709 and between electrodes at the opposite sides at distance
710 $R = \sqrt{a_0^2 + h^2}$, respectively, in one-to-one analogy with
711 the overlapping coefficients γ_1, γ_2 of the tight-binding
712 model.

713 2. Two-atom floating-potential- L crystal

714 For the case of the two-atom crystal with an alterna-
715 tion of floating-potential EBCs and inductance loads —
716 its unit cell is given in Fig. 3(b)— we proceed in a similar
717 manner to construct its equivalent circuit [see Fig. 10(b)]
718 that models the two electric resonators. Using the same
719 steps as for the previous case, we obtain, after a relatively
720 lengthy but straightforward algebra, the following
721 dispersion equation for the two electric bands

$$\omega^\pm(k_1) = \left\{ 2LC \left[\frac{C_1 + 2C_2}{C} + \frac{C_s}{C} \frac{2C + C_s}{C + 2C_s} \right. \quad (\text{A2}) \right. \\ \left. - \left(\frac{C_1}{C} + \frac{C_s}{C} \frac{C_s}{C + 2C_s} \right) \cos k_1 a \right. \\ \left. \mp 2 \left(\frac{C_2}{C} + \frac{C_s}{C + 2C_s} \right) \cos \frac{k_1 a}{2} \right] \right\}^{-1/2}.$$

722 At Γ point two distinct eigensolutions are found, one fi-
723 nite $\omega_0^- = [8LC_2(1 + \frac{C}{C_2} \frac{C_s}{2C_s + C})]^{-1/2}$ and one diverg-
724 ing as $\omega_0^+ \sim [LC_1(1 + \frac{C_s}{C_1} \frac{C_s}{2C_s + C})]^{-1/2} (k_1 a)^{-1}$. The
725 higher frequency branch $\omega^+(k_1)$ behaves as an hyper-
726 bolic function for $k_1 a \ll 1$, while the lower frequency
727 branch $\omega^-(k_1)$ corresponds to a cos-like resonant band
728 of finite width $\Delta\omega^- = |\omega_0^- - \omega_\pi|$, where ω_π is the com-
729 mon, double degenerate eigenfrequency at X point, found
730 to be $\omega_\pi = [4L(C_1 + C_2 + C_s \frac{C + C_s}{C + 2C_s})]^{-1/2}$. The form
731 of these two dispersive modes is still controlled by L and
732 the internal effective parameters of the model C, C_s, C_1

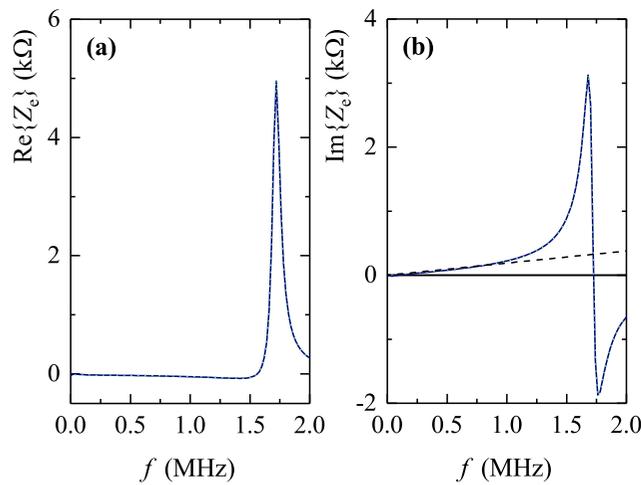


FIG. 11. Real (a) and imaginary (b) part of the impedance load Z_e used in the external circuits of Fig. 3. Solid and dotted lines represent the real (measured) and modeled [Eq. (B1)] function, respectively; in (b) the impedance of a pure inductance $L = 30 \mu\text{H}$ is also shown (broken line) for comparison.

733 and C_2 , whose physical meaning is the same as that for
734 the grounded- L crystal.

735 3. Intercellular- L crystal

736 The unit cell of Fig. 3(c) is modeled by the periodic
737 transmission line whose unit cell is shown in Fig. 10(c).
738 We easily obtain the following dispersion relation

$$\begin{aligned} \omega(k_1) &= \omega_0 \left[\frac{C}{4C_s} + \left(2 \sin \frac{k_1 a}{2} \right)^{-2} \right]^{-1/2}, \\ \omega_0 &= \left[\frac{1}{LC} \left(1 + \frac{C}{4C_s} \right) \right]^{1/2} \end{aligned} \quad (\text{A3})$$

739 from which one can immediately deduce the angular fre-
740 quency at X point, $\omega_\pi = \frac{1}{\sqrt{LC_s}}$, and the effective medium
741 velocity at the long wavelength limit $c_{\text{eff}} = \lim_{k_1 \rightarrow 0} \frac{\omega}{k_1} =$
742 $a\omega_0$. In this simplified but still quite accurate model,
743 knowledge of two external parameters, e.g. ω_π and ω_0
744 (or, equivalently, c_{eff}), is sufficient to determine the in-
745 ternal model parameters, C and C_s .

746 Appendix B: Impedance-load function

747 The external impedance load is ideally assumed to be
748 an inductance L connected to the metallic strips, as de-
749 tailed in Fig. 3. However, in practice, the real compo-
750 nent, used in our experiments, has not a pure inductive
751 behavior; other contributions have to be taken into ac-
752 count, related to several internal circuitry parts of re-
753 sistive and/or capacitive type of this component. The
754 measured impedance Z_e of our individual component — a

755 representative one, extracted from the crystal— is plot-
756 ted in Fig. 11 against frequency, f , and both its real and
757 imaginary parts [plots (a) and (b), respectively] show a
758 resonant behavior at about 1.71 MHz. Within the fre-
759 quency range that interest us here (up to 2 MHz) this
760 function can be in a very good approximation described
761 by the following rational expression

$$\begin{aligned} Z_e &= \frac{\alpha_4 f^4 + \alpha_3 f^3 + \alpha_2 f^2 + \alpha_1 f + \alpha_0}{f^2 + \beta_1 f + \beta_0} \quad (\text{B1}) \\ &= \gamma_0 + \gamma_1 f + \gamma_2 f^2 + \frac{\delta_+}{f - f_+} + \frac{\delta_-}{f - f_-} \end{aligned}$$

762 where f is expressed in MHz in the above expressions,
763 and α_n and β_n are appropriate complex coefficients ob-
764 tained by a fitting procedure

$$\begin{aligned} \alpha_4 &= -5.15566 - i5.18013 \times 10^{-14}, \quad (\text{B2}) \\ \alpha_3 &= -6.96706 \times 10^{-14} + i22.0970, \\ \alpha_2 &= 52.4644 + i4.78721 \times 10^{-13}, \\ \alpha_1 &= 4.56112 \times 10^{-13} - i452.779, \\ \alpha_0 &= 56.4767 - i7.41831 \times 10^{-13}, \\ \beta_1 &= -7.61040 \times 10^{-17} - i0.0790166, \\ \beta_0 &= -2.94215 + i8.96296 \times 10^{-18}, \end{aligned}$$

765 and

$$\begin{aligned} \gamma_0 &= \alpha_2 - \alpha_3 \beta_1 + \alpha_4 (\beta_1^2 - \beta_0) \quad (\text{B3}) \\ &= 35.5818 + i3.22798 \times 10^{-13}, \\ \gamma_1 &= \alpha_3 - \alpha_4 \beta_1 \\ &= -6.59699 \times 10^{-14} + i21.6896, \\ \gamma_2 &= \alpha_4, \\ \delta_\pm &= \pm 51.4400 - i193.077, \\ f_\pm &= \frac{1}{2} \left(-\beta_1 \pm \sqrt{\beta_1^2 - 4\beta_0} \right) \\ &= \pm 1.71481 + i0.0395083. \end{aligned}$$

766 ¹M. Leamy, M. Carrara, A. Erturk, M. Cacan, and M. Ruzzene,
767 "Metamaterial-inspired structures and concepts for elastoacous-
768 tic wave energy harvesting," *Smart Mater. Struct.* **22**, 065004
769 (2013).

770 ²J. Li, X. Zhou, G. Huang, and G. Hu, "Acoustic metamaterials
771 capable of both sound insulation and energy harvesting," *Smart*
772 *Mater. Struct.* **25**, 045013 (2016).

773 ³L. Wu, Y. Wang, K. Chuang, F. Wu, Q. Wang, W. Lin, and
774 H. Jiang, "A brief review of dynamic mechanical metamaterials
775 for mechanical energy manipulation," *Mater. Today* **44**, 168–193
776 (2021).

777 ⁴Z. Liu, X. Zhang, Y. Mao, Y.Y. Zhu, Z. Yang, C.T. Chan, and
778 P. Sheng, "Locally resonant sonic materials," *Science* **289**, 1734–
779 1736 (2000).

780 ⁵J. Li and C.T. Chan, "Double-negative acoustic metamaterial,"
781 *Phys. Rev. E* **70**, 055602(R) (2004).

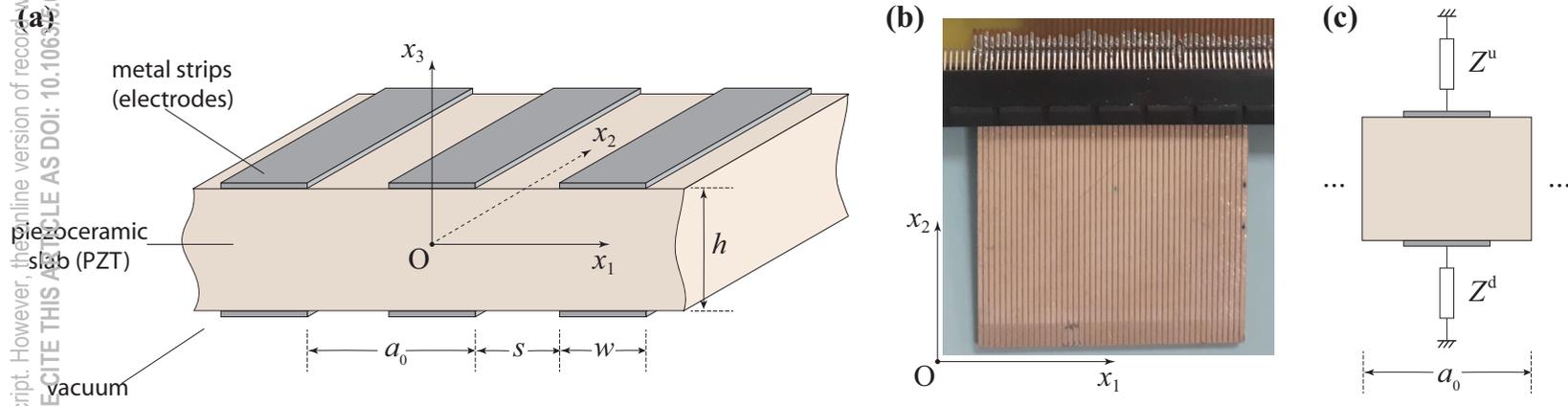
782 ⁶A. Ba, A. Kovalenko, C. Aristégui, O. Mondain-Monval, and
783 T. Brunet, "Soft porous silicone rubbers with ultra-low sound
784 speeds in acoustic metamaterials," *Sci. Rep.* **7**, 40106 (2017).

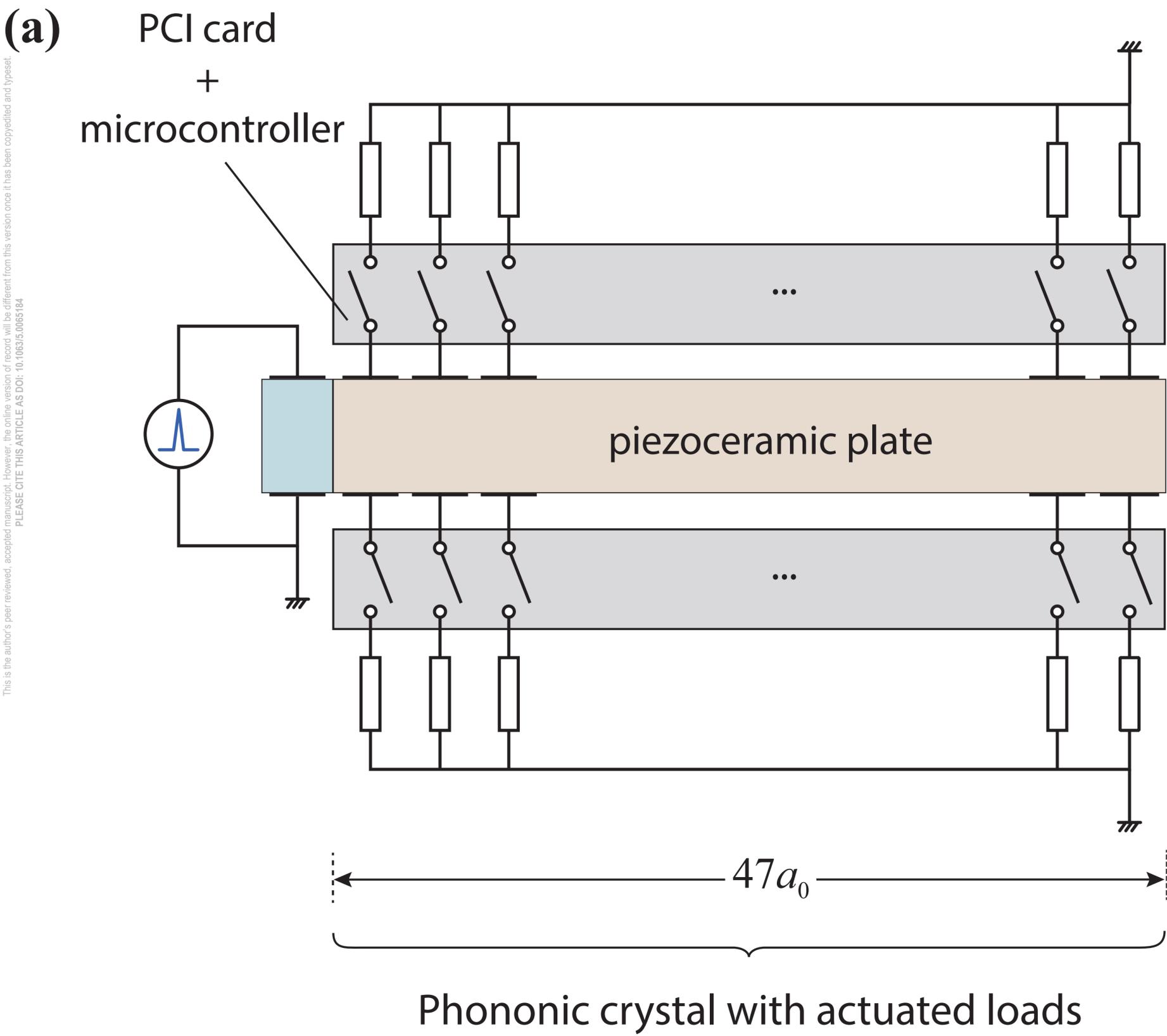
785 ⁷P. Celli, B. Yousefzadeh, C. Daraio, and S. Gonella, "Bandgap
786 widening by disorder in rainbow metamaterials," *Appl. Phys.*
787 *Let.* **114**, 091903 (2019).

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184

- 788 ⁸M. Miniaci, A. Krushynska, A.S. Gliozzi, N. Kherraz, F. Bosia, 830
789 and N.M. Pugno, "Design and Fabrication of Bioinspired Hier- 831
790 archical Dissipative Elastic Metamaterials," *Phys. Rev. Applied* 832
791 **10**, 024012 (2018). 833
- 792 ⁹Q.J. Lim, P. Wang, S.J.A. Koh, E.H. Khoo, and K. Bertoldi, 834
793 "Wave propagation in fractal-inspired self-similar beam lattices," 835
794 *Appl. Phys. Lett.* **107**, 221911 (2015). 836
- 795 ¹⁰J.-F. Robillard, O. Bou Matar, J.O. Vasseur, P.A. Deymier, 837
796 M. Stippinger, A.-C. Hladky-Hennion, Y. Pennec, and B. Djafari- 838
797 Rouhani, "Tunable magnetoelastic phononic crystals," *Appl.* 839
798 *Phys. Lett.* **95**, 124104 (2009). 840
- 799 ¹¹W. Qian, Z. Yu, X. Wang, Y. Lai, and B.B. Yellen, "Elastic 841
800 metamaterial beam with remotely tunable stiffness," *J. Appl.* 842
801 *Phys.* **119**, 055102 (2016). 843
- 802 ¹²S.W. Xiao, G.C. Ma, Y. Li, Z.Y. Yang, and P. Sheng, "Ac- 844
803 tive control of membrane-type acoustic metamaterial by electric 845
804 field," *Appl. Phys. Lett.* **106**, 91904 (2015). 846
- 805 ¹³Y. Cheng, X.J. Liu, and D.J. Wu, "Temperature effects on the 847
806 band gaps of lamb waves in a one-dimensional phononic-crystal 848
807 plate," *J. Acoust. Soc. Am.* **129**, 1157-1160 (2011). 849
- 808 ¹⁴Z. Li, Y. Li, S. Kumar, and H.P. Lee, "Thermal tuning of negative 850
809 effective mass density in a two-dimensional acoustic metamaterial 851
810 with hexagonal lattice," *J. Appl. Phys.* **126**, 155102 (2019). 852
- 811 ¹⁵E. Walker, D. Reyes, M.M. Rojas, A. Krokhn, Z. Wang, and 853
812 A. Neogi, "Tunable ultrasonic phononic crystal controlled by in- 854
813 frared radiation," *Appl. Phys. Lett.* **105**, 143503 (2014). 855
- 814 ¹⁶A.S. Gliozzi, M. Miniaci, A. Chiappone, A. Bergamini, B. Morin, 856
815 E. Descrovi, "Tunable photo-responsive elastic metamaterials," 857
816 *Nat. Comm.* **11**, 2576 (2020). 858
- 817 ¹⁷L. Airoidi and M. Ruzzene, "Design of tunable acoustic metama- 859
818 terials through periodic arrays of resonant shunted piezos," *New* 860
819 *J. Phys.* **13**, 113010 (2011). 861
- 820 ¹⁸R. Zhu, Y.Y. Chen, M.V. Barnhart, G.K. Hu, C.T. Sun, and 862
821 G.L. Huang, "Experimental study of an adaptive elastic metama- 863
822 terial controlled by electric circuits," *Appl. Phys. Lett.* **108**, 864
823 011905 (2016). 865
- 824 ¹⁹C. Sugino, M. Ruzzene, and A. Erturk, "Design and analysis 866
825 of piezoelectric metamaterial beams with synthetic impedance 867
826 shunt circuits," *IEEE/ASME Trans. Mechatronics* **23**, 2144-55 868
827 (2018). 869
- 828 ²⁰S. Degraeve, C. Granger, B. Dubus, J.O. Vasseur, M. Pham Thi, 870
829 and A.-C. Hladky-Hennion, "Bragg band gaps tunability in an 871
homogeneous piezoelectric rod with periodic electrical boundary 872
conditions," *J. Appl. Phys.* **115**, 194508 (2014). 873
- ²¹S. Degraeve, C. Granger, B. Dubus, J.O. Vasseur, M. Pham Thi, 874
and A.-C. Hladky-Hennion, "Tunability of Bragg band gaps in 875
one-dimensional piezoelectric phononic crystals using external 876
capacitances," *Smart Mater. Struct.* **24**, 085013 (2015). 877
- ²²S.A. Mansoura, P. Benard, B. Morvan, P. Marechal, A.- 878
C. Hladky-Hennion, and B. Dubus, "Theoretical and experimen- 879
tal analysis of a piezoelectric plate connected to a negative ca- 880
pacitance at MHz frequencies," *Smart Mater. Struct.* **24**, 115032 881
(2015). 882
- ²³N. Kherraz, L. Haumesser, F. Levassort, P. Benard, and B. Mor- 883
van, "Controlling Bragg gaps induced by electric boundary con- 884
ditions in phononic piezoelectric plates," *Appl. Phys. Lett.* **108**, 885
093503 (2016). 886
- ²⁴N. Kherraz, L. Haumesser, F. Levassort, P. Benard, and B. Mor- 887
van, "Hybridization bandgap induced by an electrical resonance 888
in piezoelectric metamaterial plates," *J. Appl. Phys.* **123**, 094901 889
(2018). 890
- ²⁵N. Kherraz, F.H. Chikh-Bled, R. Sainidou, B. Morvan, and 891
P. Rembert, "Tunable phononic structures using Lamb waves 892
in a piezoceramic plate," *Phys. Rev. B* **99**, 094302 (2019). 893
- ²⁶F.H. Chikh-Bled, N. Kherraz, R. Sainidou, P. Rembert, and 894
B. Morvan, "Piezoelectric phononic plates: retrieving the fre- 895
quency band structure via all-electric experiments," *Smart* 896
Mater. Struct. **28**, 115046 (2019). 897
- ²⁷We used the COMSOL Multiphysics v. 5.5 software to per- 898
form the calculations (www.comsol.com. COMSOL AB, Stock- 899
holm, Sweden). 900
- ²⁸<https://www.americanpiezo.com/>. 901
- ²⁹M. Tummala, "New algorithm for solving block matrix equations 902
with applications in 2-D AR spectral estimation," *IEEE Trans.* 903
Signal Processing **39**, 759-764 (1991). 904
- ³⁰N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (New 905
York: Saunders College Publishing, 1976). 906
- ³¹The equivalent electric circuit that models this unit cell coincides 907
to that shown in Fig.A1(b) of Ref. 26, and the dispersion relation 908
of this mode is still given by Eq.(2) of the above-mentioned 909
reference. 910
- ³²R. Sainidou, N. Stefanou, I.E. Psarobas, and A. Modinos, "Scat- 911
tering of elastic waves by a periodic monolayer of spheres," *Phys.* 912
Rev. B **66**, 024303 (2002). 913

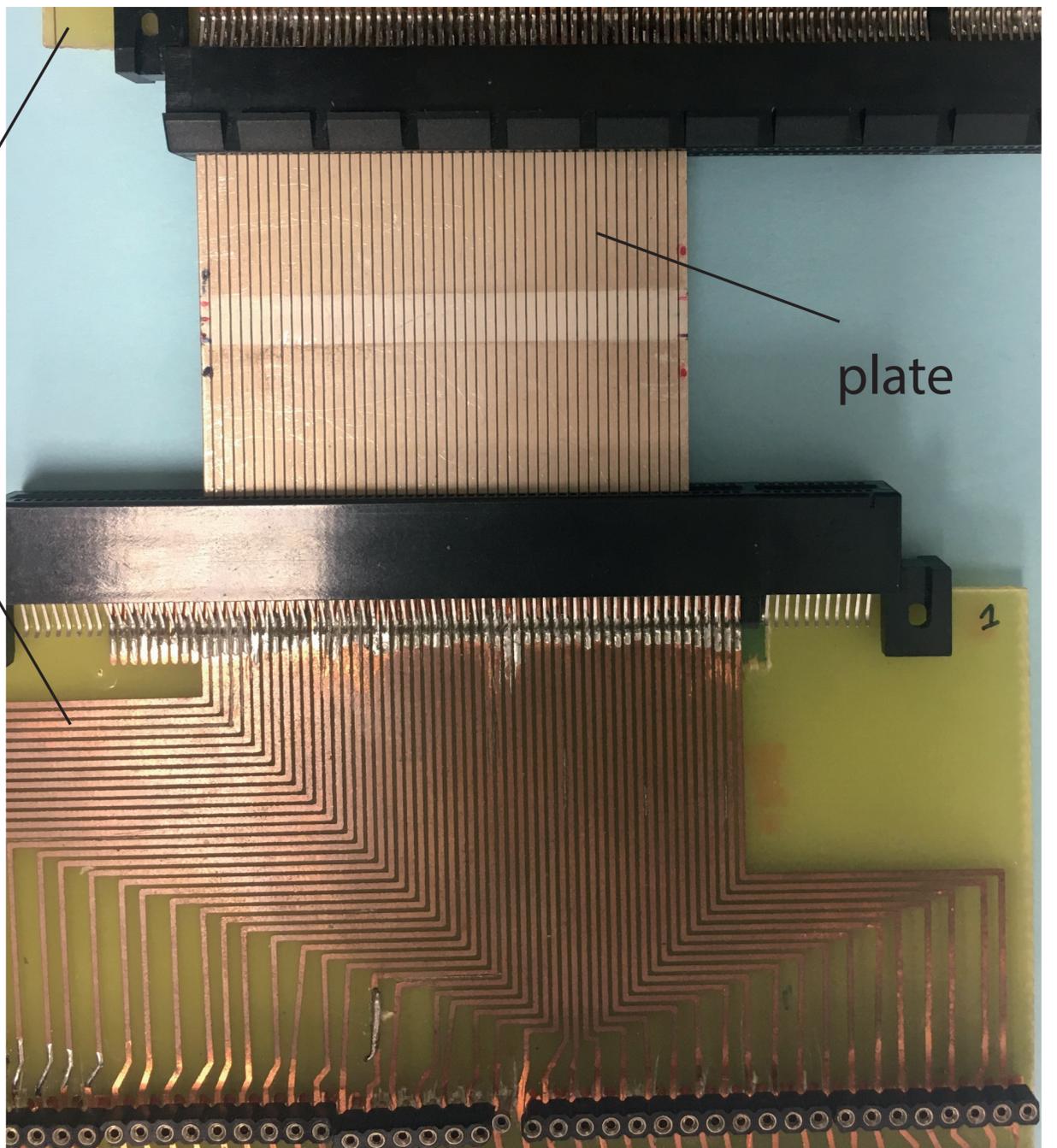
This is the author's peer reviewed, accepted manuscript. However, the online version of records will be different from this version once it has been copyedited and typeset.
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/1.50065184



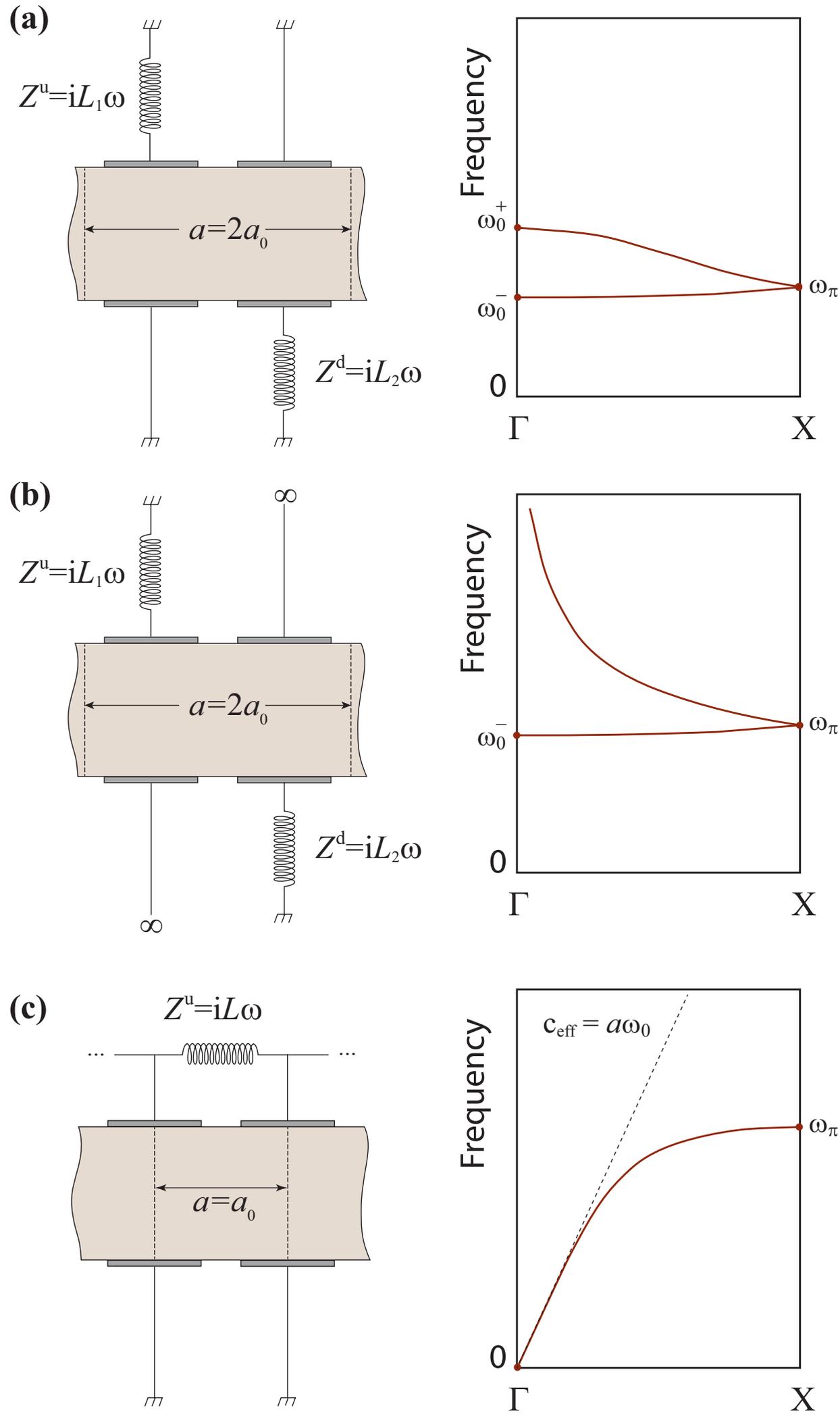
**(b)**

PCI cards

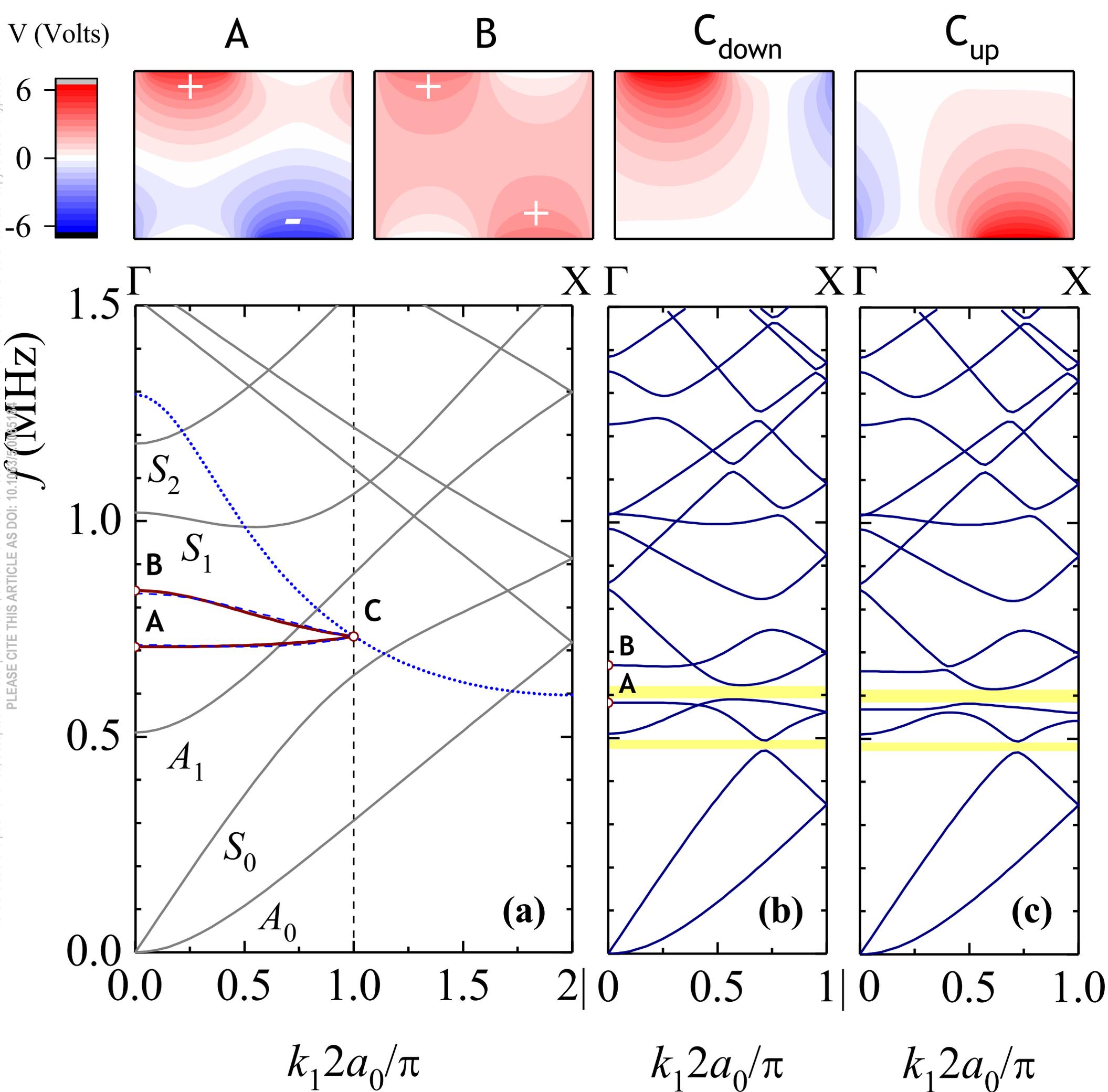
plate



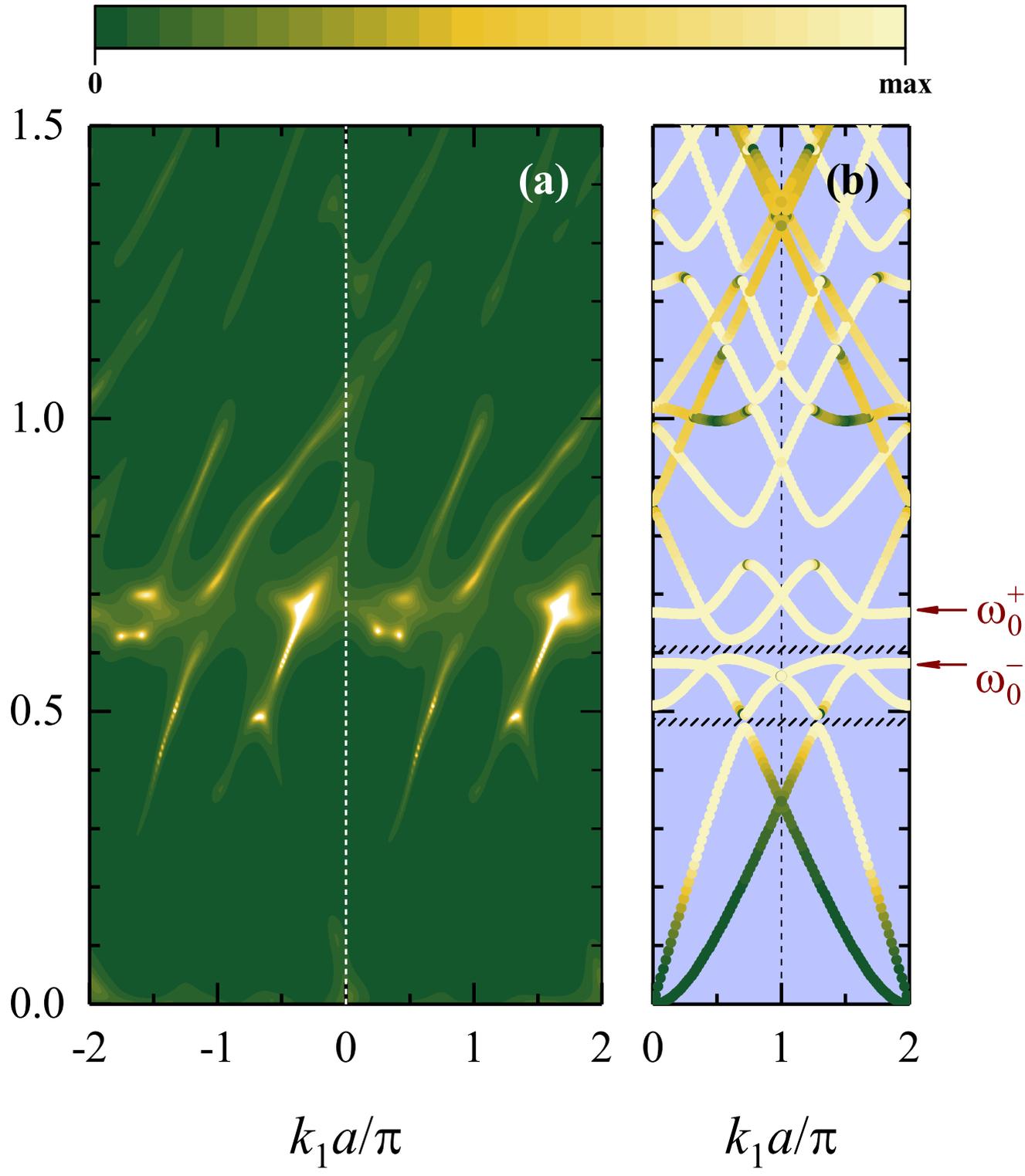
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184



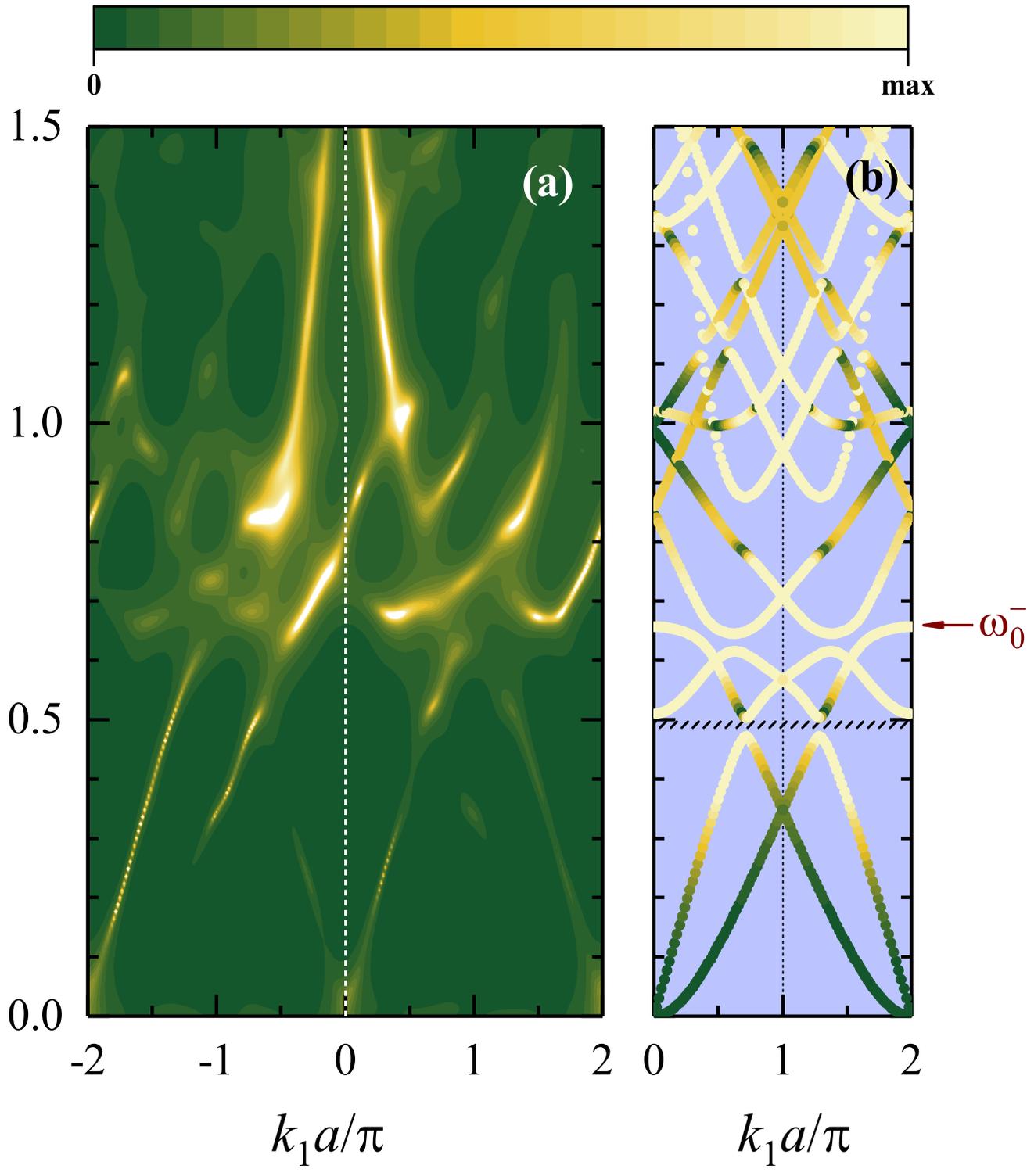
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset. PLEASE CITE THIS ARTICLE AS DOI: 10.1063/1.5003514



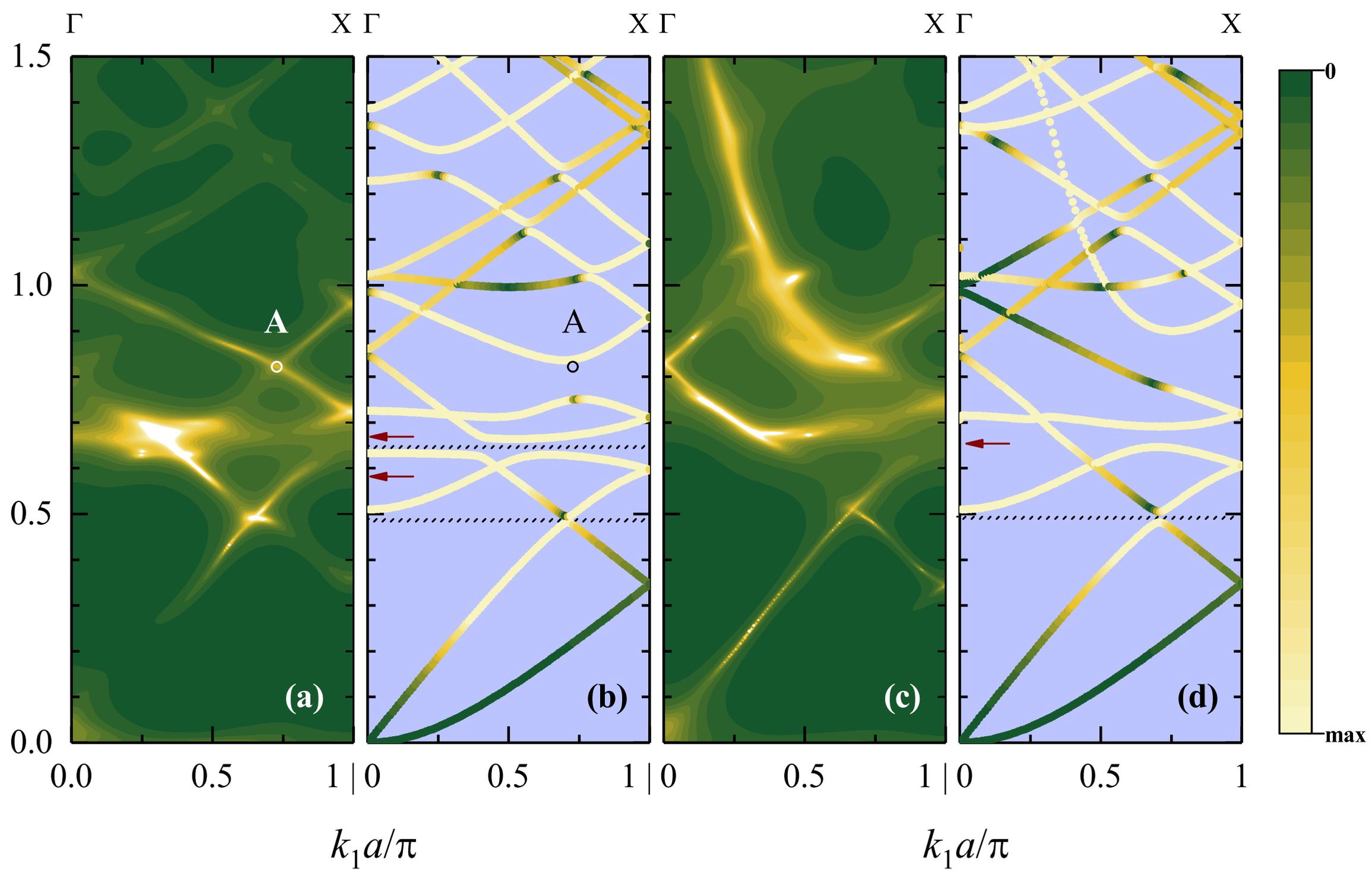
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184

 f (MHz)

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184

 f (MHz)

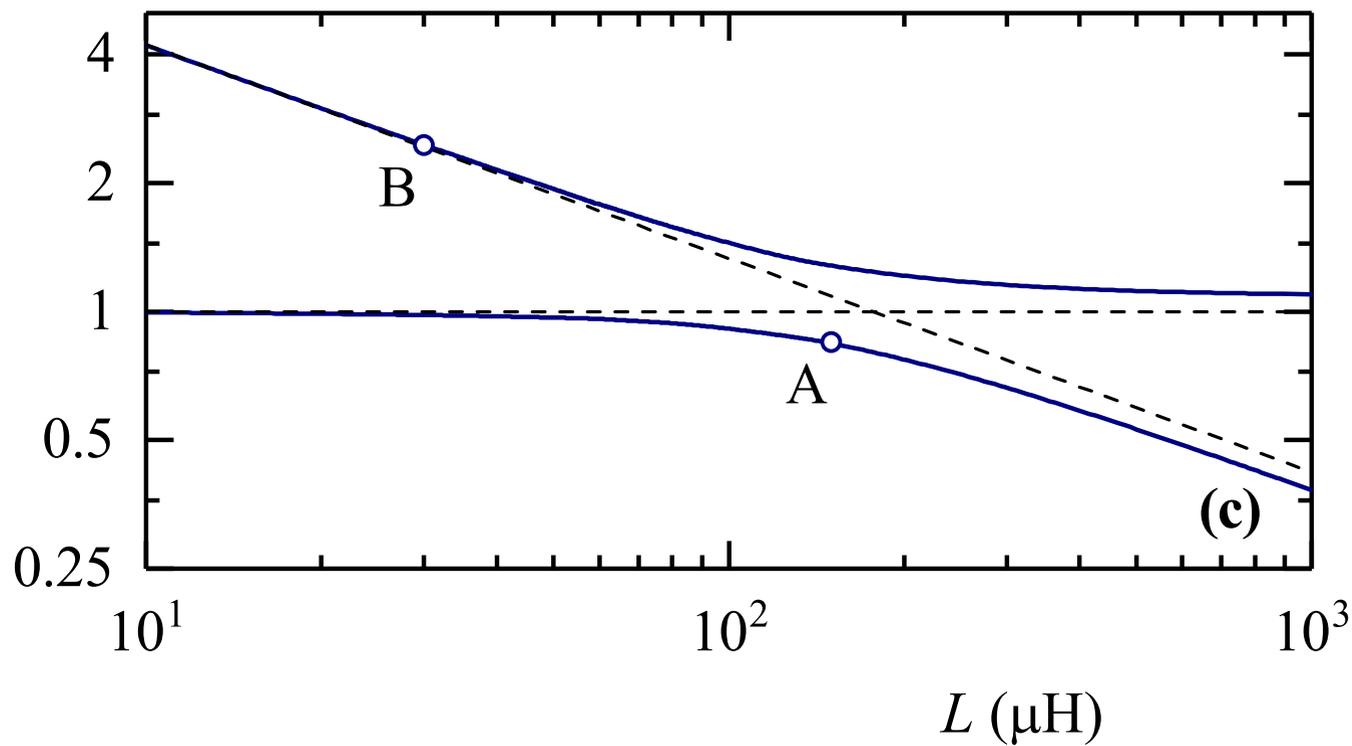
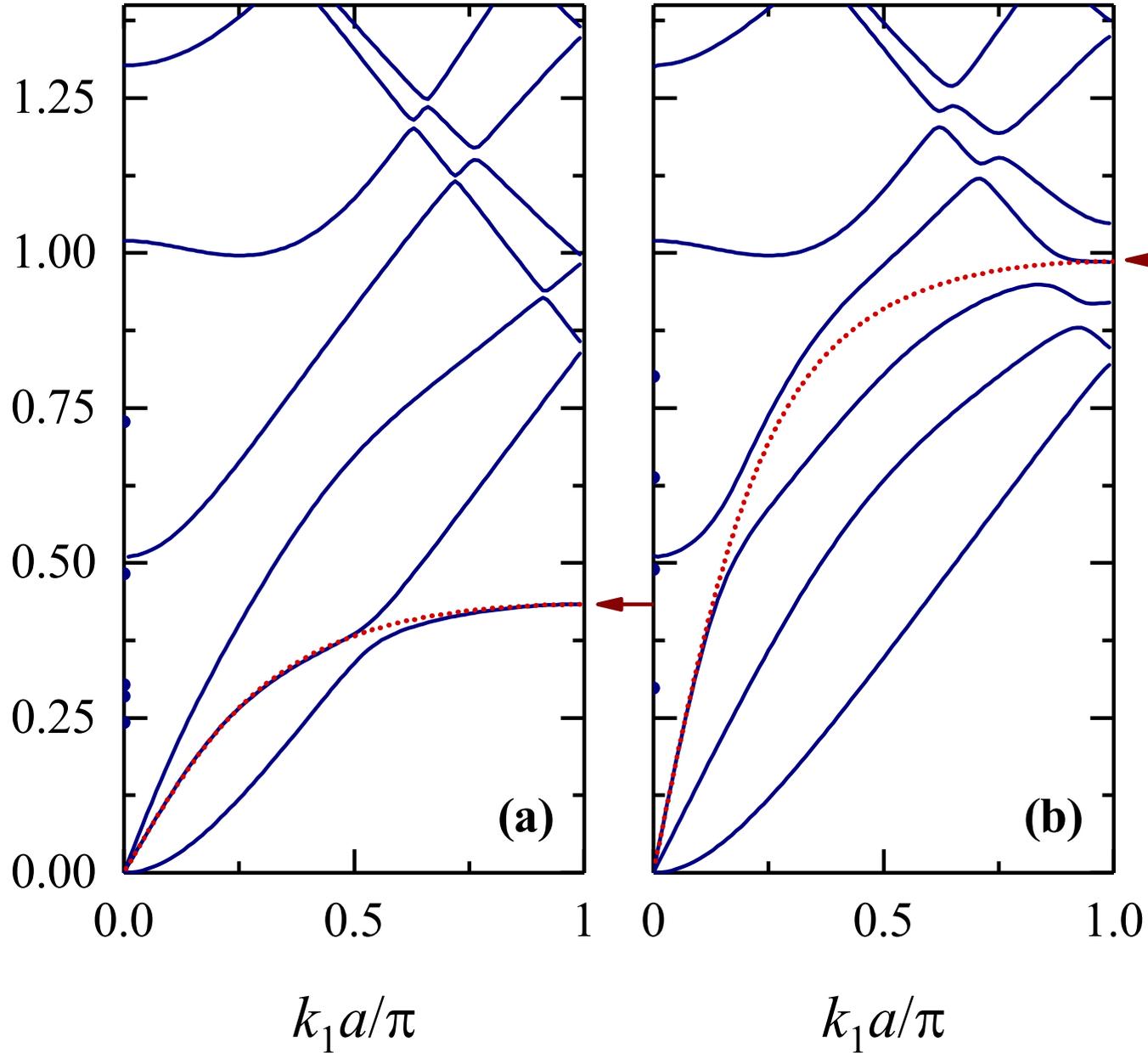
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184



This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184

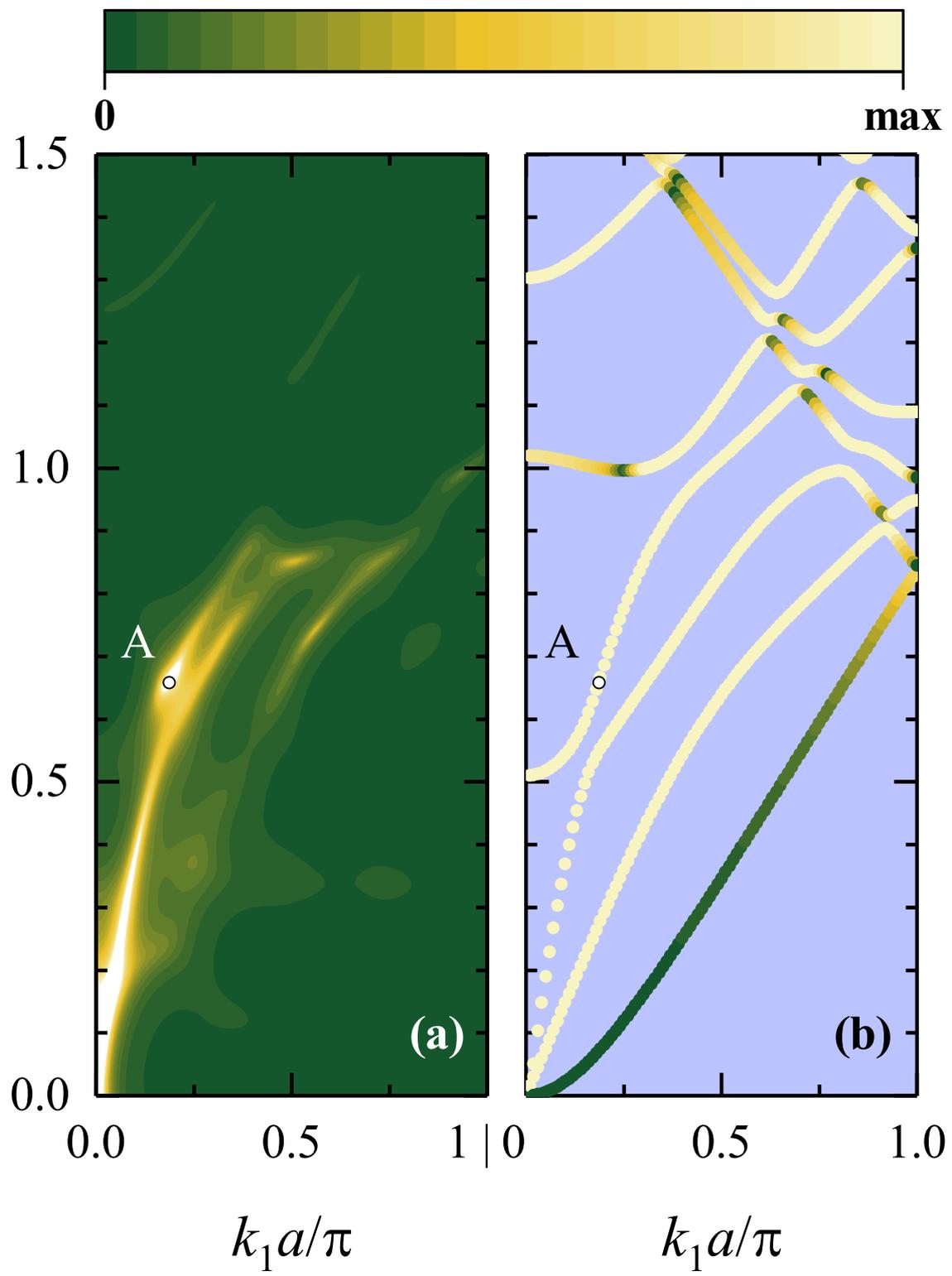
f (MHz)



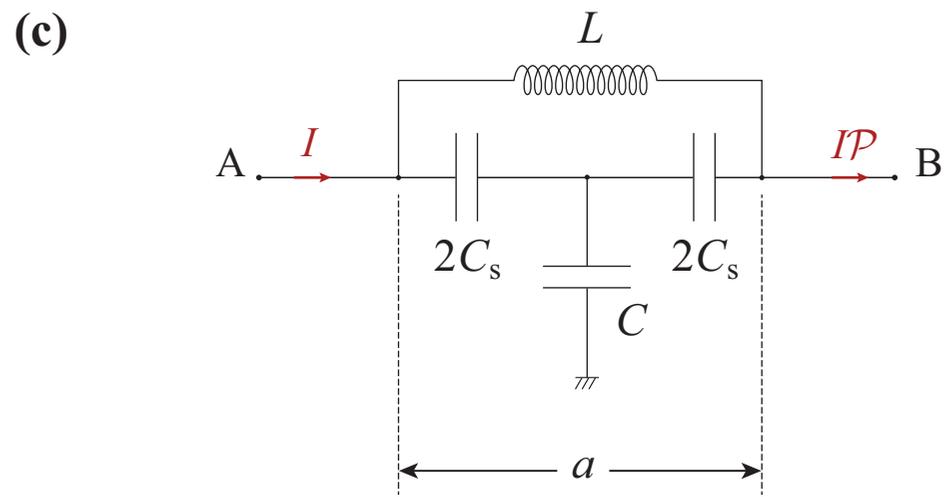
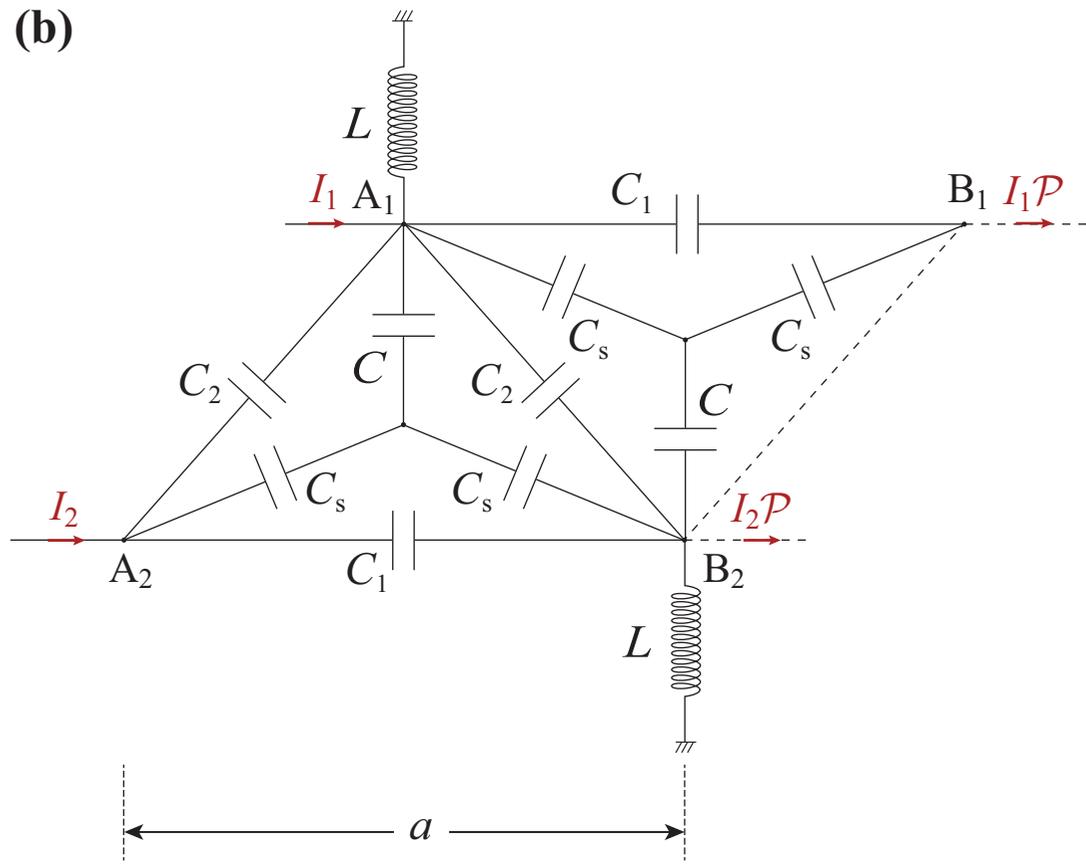
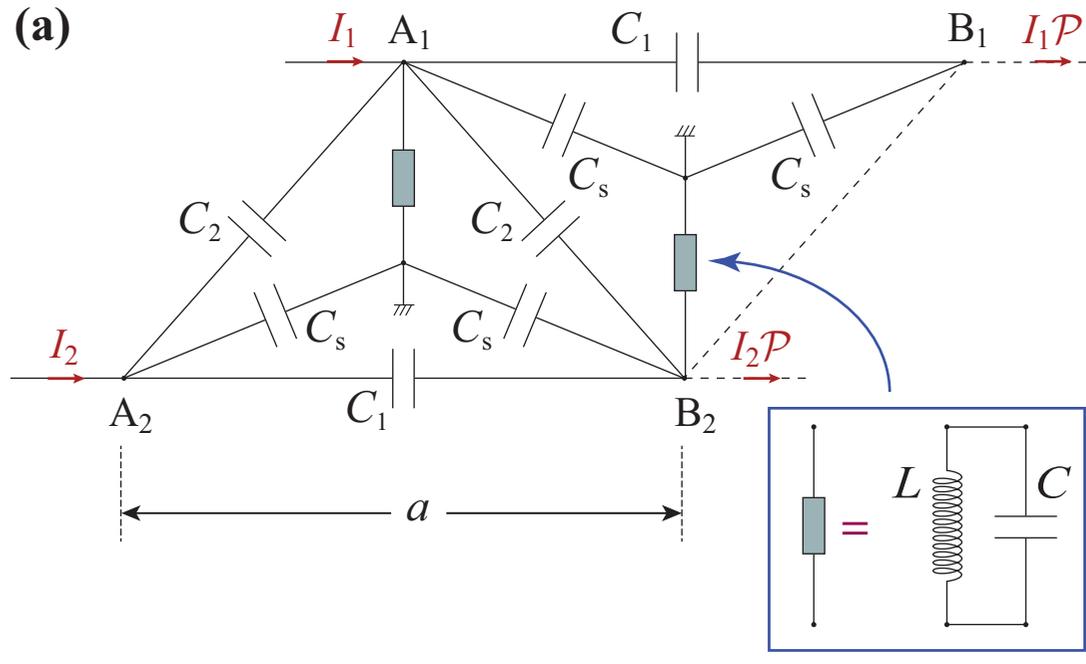
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184

f (MHz)

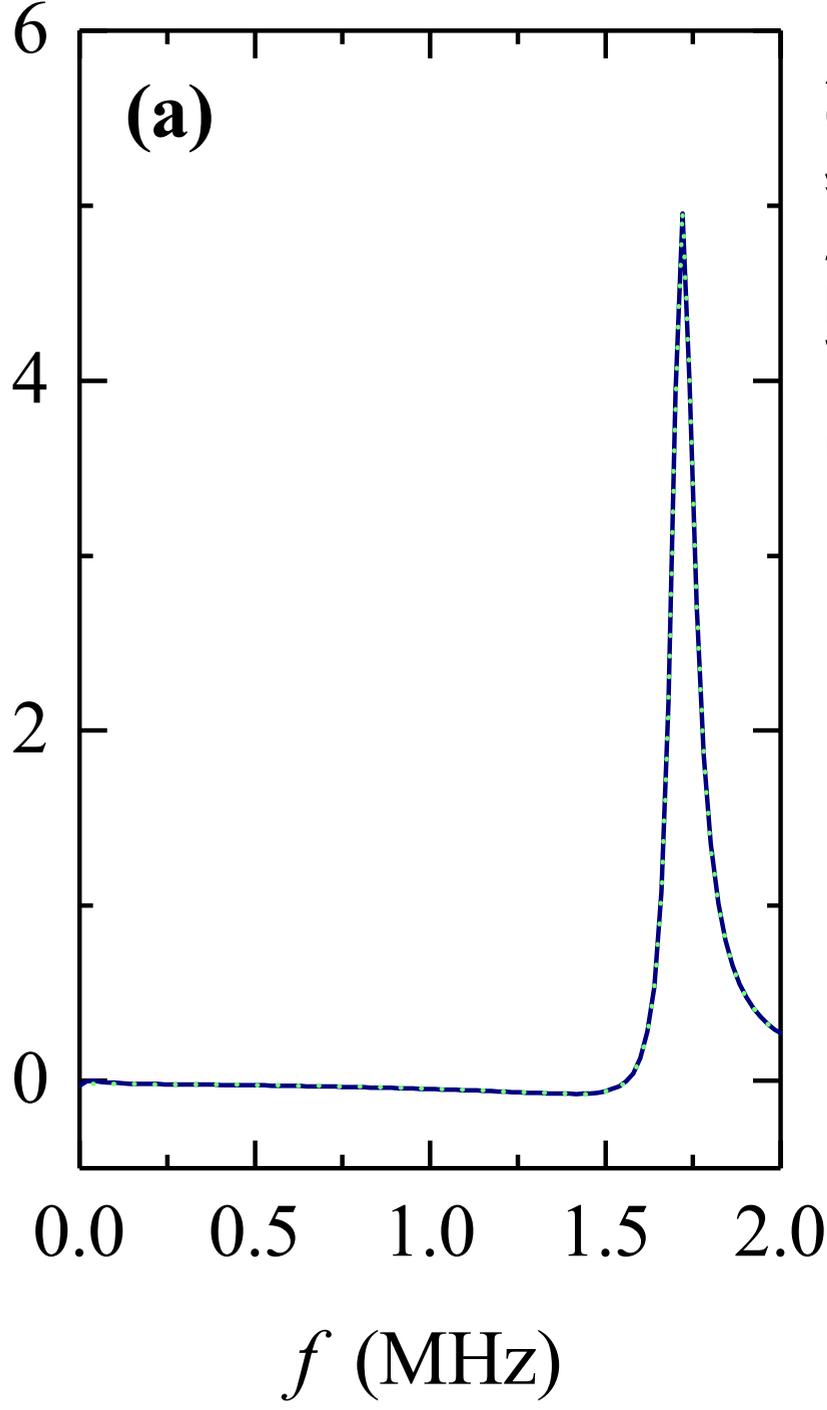


This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset. PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184



This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0065184

$\text{Re}\{Z_e\}$ (k Ω)



$\text{Im}\{Z_e\}$ (k Ω)

