# Ibrāhīm al-Balīshṭār's book of arithmetic (ca. 1575): Hybridizing Spanish mathematical treatises with the Arabic scientific tradition

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**Abstract**

We present an Arabic treatise on arithmetic, penned in Cherchell (Algeria) around 1575, of which five manuscripts are extant. The author, Ibrāhīm al-Balīshṭār, is a hitherto unknown Morisco mathematician from Aragon. After sketching his biography, we show that his treatise, claimed by him to be the translation of a book by a Christian priest, is actually an elaborate personal work, resulting from the intertwining of two Spanish treatises–those by Marco Aurel and Juan de Ortega–and of material drawn from Arabic authors such as al-Ghurbī, al-ʿUqbānī and Ibn al-Bannāʾ. It is therefore an original attempt to create a Euro-Islamic hybrid knowledge.

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Nous présentons un traité d'arithmétique en arabe, rédigé à Cherchell (Algérie) vers 1575, dont cinq manuscrits subsistent. L'auteur, Ibrāhīm al-Balīshṭār, est un mathématicien morisque jusqu'ici inconnu, originaire de l'Aragon. Après avoir esquissé sa biographie, nous montrons que son traité, qu'il dit être la traduction d'un livre d'un prêtre chrétien, est en réalité un travail personnel élaboré, résultant de l'entrelacement de deux traités espagnols – ceux de Marco Aurel et Juan de Ortega – et d'éléments tirés d'auteurs arabes comme al-Ghurbī, al-ʿUqbānī et Ibn al-Bannāʾ. Il s'agit donc d'une tentative extraordinaire de création d'un savoir hybride euro-islamique.

*MSC:* 01A30, 01A40

*Keywords:* Moriscos; translation; arithmetic; Marco Aurel; Juan de Ortega; Muḥammad al-Ghurbī.

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DECLARATION OF INTEREST: NONE.

THIS RESEARCH DID NOT RECEIVE ANY SPECIFIC GRANT FROM FUNDING AGENCIES IN THE PUBLIC, COMMERCIAL, OR NOT-FOR-PROFIT SECTORS.

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## Introduction

Over the past few years, some attention has been paid to early stages of the transfer of modern European science to Islamic countries.[1] Much of the research has focused on translation, which represents the most tangible evidence of knowledge transfer. Most of the time, little is known about the translators. What can be gleaned from sources reveals a wide range of cultural go-betweens with diverse backgrounds and experiences: travellers, merchants, missionaries, converts, captives, exiles, interpreters, native scholars, translation pairs consisting of a native and a non-native, and so forth. The case reported in this article goes back to the sixteenth century: it is the earliest encounter we know of between Islamic mathematics and European mathematics, two long-separated sister traditions. Its specificity lies in the fact that the mediator attempted to create a hybrid between the two.

The origin of our research goes back to 1986, when one of us (H. H.) discovered at the National Library of Tunisia a previously unknown Arabic language arithmetical treatise, containing many original problems on commercial transactions. He immediately noticed an unusual and intriguing fact: the author of the treatise, a certain Ibrāhīm al-Balīshṭār living in Cherchell (Algeria), claims to have translated a book by a Christian priest. He analyzed the mathematical content of the treatise in some detail, but other occupations prevented him from publishing his observations, which remained in the form of handwritten notes in Arabic. It was not until 2018 that he announced this discovery at the *Colloque maghrébin d'histoire des mathématiques arabes* in Tunis (Hedfi 2019) and, simultaneously, in the catalogue of a collection of scientific manuscripts kept in the National Library (Abdeljaouad and Hedfi 2018, 111-115, 148-152). At this point, the author of the treatise remained mysterious and his sources unidentified. During the *Colloque maghrébin*, we decided to combine our efforts towards a better understanding of this intriguing treatise. This article sums up the main results of our joint research. Sections 1 and 2 cover the context in which al-Balīshṭār's treatise was written and what we could determine about its author. Section 3 describes its five extant copies, with a tentative *stemma codicum*. Sections 4 and 5 discuss its European and Arabic sources. Section 6 describes its contents, with special focus on two chapters which seem to us to be representative of the whole project. The article rounds out with an epilogue aimed at clarifying some confusion in historiography.

We are currently preparing a complete critical edition of al-Balīshṭār's treatise, including a French translation and extensive historical and mathematical comments. The reader is referred to this edition for more details.

[1] See for examples (Günergun 2007), (Abdeljaouad 2011), (Ageron 2019).

## 1. Overview of the Aragonese Morisco community in the sixteenth century

As will turn out in the next section, the author of the arithmetical treatise we discovered in Tunis was born in the Morisco community living in Aragon in the sixteenth century. This community is not as well-known as its analogues in the southern regions of Spain, because few traces of its past were known until recently. We felt it would be useful to sum up the picture that has emerged from scholarly research.[2]

After the progressive Christian reconquest of Aragonese cities (Huesca in 1096, Barbastro in 1102, Saragossa in 1118, Teruel in 1171), an important Muslim population remained in Aragon for five centuries. According to a census taken in 1495, Muslim households made up eleven percent of the total number.[3] Some Muslim families lived in specific neighbourhoods of the cities, called *morerías*; their way of life was very similar to that of the Christians. However dozens of isolated villages were inhabited solely by Moors who firmly retained Muslim customs and practices. Each village had its own mosque; on important occasions, the local Islamic community (*aljama de los moros*) gathered in the forecourt. Most villagers worked as farmers on behalf of the Christian landowners, or as craftsmen. Whereas the men had a good command of Aragonese for contacts with Christians, dialectal Arabic remained the everyday domestic language. Literal Arabic was taught when possible, but young men wishing to study it in depth were sent to the neighbouring Kingdom of Valencia, where there were more teachers. Intellectual life was far from inexistent: in fact, Aragonese Muslims living in small rural communities had an amazing love of books. A cultural elite made up of *alfaqís*, judges, notaries and physicians was literate in both Castilian and Arabic. Besides the Quran, some Islamic law compendia and medical treatises in Arabic circulated and were actively copied and studied. From 1450 onwards, this elite also produced *aljamiado* literature, i.e. Castilian or Aragonese texts in Arabic script.

Christian lords appreciated the labour force provided by their Muslim vassals. But the uprising of the lower Christian classes of Valencia known as "Revolt of the Brotherhoods" (*Germanías*) prompted Charles V to issue a forced conversion edict: by 31th January 1526, every Muslim of the territories ruled by the Crown of Aragon, i.e. Aragon proper, Catalonia and Valencia, had to become a Christian or to leave. After a while, it became obvious that a majority of the so-called New Christians, or Moriscos, still practiced Islam clandestinely (Harvey 2005, 49). In 1564, Arabic language was banned, and the pressure of the Inquisition tribunal increased. Hundreds of trials were

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[2] Two major references on Muslims in Spain during the sixteenth century are (Harvey 2005) and (Vincent 2017). On the linguistic practices and the intellectual life among Aragonese Moriscos, we also used the following sources: (Ribera 1925), (Fournel-Guérin 1979), (de Epalza 1992, 111-118), (García-Ballester 1994), (Miller 2008, ch. 3), (Ruiz-Bejarano 2015), (Bernabé-Pons 2017).

[3] Precisely 5674 from a total of 51540. This 11% rate was probably underestimated and might have reached 15% or 20% (Colás Latorre 2010, 35).

held against Moriscos, especially against those who had been proved to own Arabic books. The failure of this forced integration policy resulted in the general expulsion of Moriscos, decided on 29th May 1610. However, a proportion of them, especially among the elite, had already fled persecution and voluntarily moved overseas, mostly to North Africa, before this final expulsion.[4]

## 2. Ibrāhīm al-Balīshṭār, from the Kingdom of Aragon to the Regency of Algiers

The author of our arithmetical treatise gives his full name as "Ibrāhīm, son of ʿAbdallāh, son of Muḥammad al-Balīshṭār al-Thaghrī"; he adds that he is "from Barbwāsh by birth and from Sharshāl by domicile and residence".[5] This is basically all what we know about him. His name is absent of all bio-bibliographical dictionaries of Maghrebian celebrities available to us.

Let us first examine the *nisba* (ethnic name) al-Thaghrī. It literally means the marchman and generally speaking refers to someone originating from the border regions of the Islamic Empire. In the context of former Islamic Spain, it was especially used in reference to the regions ruled by the Crown of Aragon. It could either denote a Muslim inhabitant of these regions or one who had left them, willingly or not, and settled somewhere else, mostly in North Africa. The Spanish word *Tagarino* and the French word *Tagarin*, derived from *Thaghrī*, always have the latter meaning.

Identifying *Barbwāsh,* our author's place of birth, was something of a puzzle. We are by now firmly convinced that it is Barbués, in the province of Huesca (or Upper Aragon), a tiny rural village dominating the valley of río Flumen at an altitude of 361 m and offering a beautiful view of the neighbouring Pyrenees.[6] Nearly opposite on the other bank of the río Flumen lies another village called Torres de Barbués. During all of the sixteenth century, Barbués and Torres de Barbués were inhabited solely by Muslims.

Our identification of *Barbwāsh* as Barbués is corroborated by a mortgage loan contract dated July 1496, signed by all members of the Moorish communities of Barbués and Torres de Barbués (Lleal & Arroyo Vega 2007, 398-399).[7] In Barbués, one of the signatories is a certain *Mahoma de Bellestar*, whom we can safely identify as Muḥammad al-Balīshṭār, our mathematician's grandfather. He is listed last, which suggests that he was at the time the youngest or the most

[4] The following example is particularly relevant for our context. Around 1570, Juan Çafar, a learned Morisco citizen of Huesca, fluent in Arabic, moved his residence to Algiers to be able to lead a life according to his faith. He never settled back in Spain, although he occasionally returned clandestinely. His house in Algiers became a rallying point for relatives and visitors (Conte Cazcarro 2009).
[5] *Ibrāhīm bin ʿAbdallāh bin Muḥammad al-Balīshṭār al-Thaghrī Barbwāshī mawlid$^{an}$ Sharshālī dār$^{an}$ wa-maskan$^{an}$.* One manuscript gives al-Balishṭār and another one al-Balshāṭir: these forms are probably errors.
[6] One of us visited Barbués on 4 July 2018.
[7] Primary source: Archivo de la Corona de Aragón, Diversos/Patrimoniales, *Fondo Sástago*, I (ligarzas), pergamino 259.

recently settled head of household of the village.[8] By contrast, a *Caci* (Qāsim) *de Bellestar* is listed first among signatories of Torres de Barbués and reported as *jurado* (a municipal officer): it may well be that he was Mahoma's father.[9] In addition to this document, we have detailed census records for Barbués and Torres de Barbués, dated November 1495. While the household of Caci de Bellestar does appear in the census record for Torres de Barbués, no Bellestar was registered in Barbués (Serrano Montalvo 1995, 136-137). This confirms that Mahoma settled in Barbués between November 1495 and July 1496. The surname *de Bellestar*, arabized as *al-Balīshṭār*, probably indicates that the family originated from Bellestar de Flumen, another entirely Muslim village in the immediate vicinity of the city of Huesca.[10]

Barbués and Torres de Barbués were–and still are–rural villages where agriculture was the main activity: from 1495 onwards, the land owner for both villages was Don Blasco de Alagón, señor and later count of Sástago.[11] Some villagers however practised other professions: for example, a medical doctor, Çalema (Salāma) Antillón, was active in Barbués around 1450 (Benedicto Gracia 2008, 67). In 1488, Barbués had a population of 19 households, all Muslim; in 1495, this number reached 21. Then it was seemingly impacted by depopulation, since only 15 families were expelled in 1610 (Reglá 1964, 43). The same phenomenon can be observed in Torres de Barbués: 16 households existed in 1488, 20 in 1495, but only 9 families were expelled in 1610. This situation is in fact exceptional and contrasts sharply with most other purely Muslim Aragonese villages, where the number of households seems to have generally doubled or tripled between 1495 and 1610 (Ubieto Arteta 1984-1986). Our hypothesis is that several families living in Barbués or Torres de Barbués fled persecution and emigrated voluntarily to North Africa before the final expulsion. Such was probably the case of our Ibrāhīm: we estimate his birth around 1530 and his departure about 1570. Like many other *Tagarinos* for over a century, he settled in Cherchell (*Sharshāl*), a coastal city 100 km west of Algiers. There, eager to revive arithmetic in Islamic countries, he decided to compose a treatise on that subject. This is seemingly the only book he authored, and he seems to

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[8] We reproduce the complete list of signatories for Barbués: *Nos, Juce Maxcarro, Ali de Maxcarro, jurado, Mahoma La Reyna, lugartenient de jurado por Mahoma Ezmel jurado del dicho lugar de Barbues, Yuce Ylel, Mahoma Zebin, Ali de Rechol, Lop Del Alcaci, Ylel de Maxcarro, Muça Antillon, Ali de Albiol, Mahoma Alfaqui, Mahoma Serrano, Ali de Calbo, Juce Almorci, Brahem de Alcaci, Mahoma Tamen et* ***Mahoma de Bellestar****, moros, vezinos del dicho lugar de Barbues.*

[9] We reproduce the complete list of signatories for Torres de Barbués: *Nos,* ***Caci de Bellestar****, Audalla de Cety, jurados del dicho lugar, Mahoma Cormano, Audalla de Camino, Çalema Antillon, Mahoma Antillon, Brahem Antillon, Exea Antillon, Brahem Ylel, Lop de·la Almunia, Audalla Fragues, Mahoma Rami, Audalla de Rapa, Çalema Palacio, Mahoma Palacio, Mahoma Al Caci, Brahem de Palacio, Brahem de Caci, Brahem d'Antillon, Mahoma Çeyt, moros vezinos de·la dicha aljama de moros del dicho lugar de Torres de Barbues.*

[10] Similarly, the census records mention in Barbués and Torres de Barbués members of a Antillón family. Antillón is the name of yet another village in the same area, from which Muslims had been expulsed in the thirteenth century (Peiró Arroyo 2008, 13).

[11] The counts of Sástago had a castle-palace built in Barbués which can still be seen at an extremity of the village. It may be dated from the beginning of the sixteenth century.

have not completed it. Thereafter, we lose track of him entirely. Did he ever try to return to Aragon?

Let us now translate the background of his book project, as narrated by him in the prologue:

> When I was in the land of Christians–may God most High destroy them and annihilate them–, I came across a book on the art of arithmetic composed by a priest (*qiṣṣīṣ*) named Almān. I studied part of it under the guidance of our master (*shaykh*), the perceptive, the clever arithmetician, the specialist of division of inheritances Sīdī Muḥammad al-Andalusī al-Gharnāṭī, called Marūkān–may God sanctify his soul and pour out His mercy on his remains. I felt it to be of the greatest utility possible, since it contains wonderful approaches and easily understood high-quality summaries, not found in other treatises on the arithmetical art, and God most High enabled me to understand it. Then I set foot on this Islamic shore: I found that this science was on the verge of disappearing completely. I did not find anybody doing well in this occupation. On the contrary, it had become the field of what is erased and lost. This raised in me a great dismay and an intense and formidable jealousy, because this science is the basis of the science of division of inheritances: indeed, it teaches how to fix them properly and how to share them (…) Then I felt that I should copy the aforementioned book, translate it from the foreign language to Arabic, devote all my efforts to it and, in some places, add to it warnings and useful things that God enabled me to master. With God's help, its utility will be increased and its rules will be made easier for students.

Later on, he narrates a story told to him by a jurist (*faqīh*), according to which a Jew was consulted for solving inheritance problems according to the Islamic law, because no Muslim was able to cope with the complicated fractions involved. He comments:

> When the *faqīh* mentioned this story to me, I was struck by a great dismay. This was one of my motivations to compose these leaves about this science, despite my ignorance of the Arabic language. But the work will be salutary with the help of God Most High.

We postpone identification of the priest Almān and his book to section 4 of this article. Concerning Ibrāhīm's *shaykh*, his Arabic name Muḥammad al-Andalusī al-Gharnāṭī indicates that he was not an Aragonese, but an Andalusian from Granada (*Gharnāṭa*). His Spanish *apellido*, rended in Arabic as *Marūkān*, is most assuredly Marugán, derived from a common toponym in Spain: for example, there is a place called Pago de Marugán in Atarfe, close to Granada.[12] The question arises as to where and when the young Aragonese man benefited of his teaching. We have no firm answer, but it seems quite likely that this happened in Valencia in the third quarter of the sixteenth century–1552 being a *terminus postquem* as we shall see.

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[12] The toponym *Marugán* is itself is a corruption of the Arabic name *Marwān.*

### 3. The manuscripts of Ibrāhīm al-Balīshṭār's arithmetical treatise

We had access to four copies of Ibrāhīm al-Balīshṭār's work. All of them are now kept in the National Library of Tunisia; however, they once belonged to three among the several libraries whose holdings were merged in 1968, pursuant to a decree issued on 7 September 1967:

– the Bibliothèque publique de la régence de Tunis, created on French initiative by a decree issued on 8 March 1885 by bey ʿAlī III, renamed Bibliothèque nationale de Tunisie in 1956: although mostly formed of French books, it had acquired many Arabic manuscripts over the years;[13]

– the earlier Aḥmadiyya Library, founded in 1840 by bey Aḥmad I within the al-Zaytūna great mosque;

– the Khaldūniyya Library, established in October 1901 within the eponymous modern school.[14]

We shall denote our four manuscripts by the sigla **A**, **B**, **C**, **D**. Here is a brief description of each of them:

**A**. National Library of Tunisia, ms. 16450/1

This is the first in a collection of three mathematical texts given in *ṣafar* 1291 / March 1874 by Muḥammad al-Ṣādiq Bāy to the Aḥmadiyya Library, where its shelfmark was 5462. The manuscript is undated. It has 52 leaves, measures 15.5 cm by 21 cm and displays 25 lines per page written in Maghribī script in an average and very legible hand. The name of the copyist is not mentioned.

**B**. National Library of Tunisia, ms. 13053/19

This is the antepenultimate in a collection of twenty-one mathematical texts that once belonged to the Aḥmadiyya Library, where its shelfmark was 6259. The manuscript is undated; however, the sixth text of the collection was copied in 1184/1770 and the thirteenth in *ramaḍān* 1184 / January 1771. It has 26 leaves, measures 14.5 cm by 20.5 cm and displays 23 lines per page written in Maghribī script in a neat and legible hand. The name of the copyist is not mentioned.

**C**. National Library of Tunisia, ms. 16497/1

This is the first in a collection of five mathematical texts given to the Khaldūniyya library at the time of its creation, in 1319/1901–as witnessed by stamps on fol. 1a, 1b and 53b. The manuscript is undated; however, the second text of the collection was copied in 1279/1863, the third in 1277/1860 and the fifth in 1269/1853. It has 54 leaves, measures 14 cm by 21 cm and displays 19 lines per

[13] The Bibliothèque publique had 800 Arabic manuscripts in 1937 (Châtelain 1937, 30), 3200 in 1949 (Rousset de Pina 1949, 58) and 4583 in 1965 (BNT 1970).

[14] The shelfmarks of the manuscripts already in National Library before 1967 range from 1 to 4900. Those now numbered 4901 to 10023 belonged before 1967 to the Ṣādiqiyya Library (founded in 1875, also within the al-Zaytūna great mosque), manuscripts 10024 to 16496 belonged to the Aḥmadiyya Library and manuscripts 16497 to 16664 belonged to the Khaldūniyya Library.

page written in Maghribī script in a rapid yet elegant hand. The name of the copyist is not mentioned.

**D**. National Library of Tunisia, ms. 00692

This is an isolated text which was previously part of a collection, as evidenced by a second foliation in Eastern Arabic numerals running from 174 to 249 (only 177 to 244 are visible). Before 1956, and probably since the 1930's, it belonged to the Bibliothèque publique de Tunis, as witnessed by a stamp on f. 13a. Its earlier location is unknown. The manuscript is undated. It has 78 leaves, measures 17 cm by 22 cm and displays 17 lines per page written in Maghribī script in a rather messy hand. The name of the copyist is not mentioned.

We compared the four manuscripts. Many obvious errors or omissions are common to all of them, which demonstrates that they ultimately derive from one common ancestor.[15] Numerous textual variants distinguish **C** from the other copies, so that **A, B** and **D** must have had a common ancestor from which **C** does not derive.[16] Further variants distinguish **A** and **B** on the one hand, **C** and **D** on the other, so that **A** and **B** must have had a common ancestor from which **D** and **C** do not derive.[17]

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[15] By way of examples, let us mention some errors or omissions common to all four manuscripts. On two occasions, exactly at the same place in all manuscripts, a space was left blank mid-sentence where missing words should have been (**A**, f. 9b and 32b, **B**, p. 297 and 316, **C**, f. 11a and 35a, **D**, f. 14a and 49a). In an alligation problem involving four ingots whose weight and fineness are given, borrowed from Ortega's book [Ortega 1552, 172a-173a] and referred to as R4 in section 6 below, the fineness of the third ingot and the weight of the fourth were omitted, probably because a copyist's eye skipped by homeoteleuton from one instance of *wa-tisˁ awāq*$^{in}$ [and nine ounces] to another ; moreover, in the solution of this problem, 284 instead of 264 is given as the result of the multiplication of 66 by 4, although the final result is correct (**A**, f. 32b, **B**, p. 316, **C**, f. 35b, **D**, f. 49b). Al-Balīshṭār's "marvellous shortening" of the rule to find the sum of consecutive terms in a geometric progression (see section 6) is unintelligible as it stands, because part of the sentence was omitted by homeoteleuton from one instance of *illā al-wāḥid* [minus one] to another (**A**, f. 37a, **B**, p. 322, **C**, f. 39a, **D**, f. 54b).

[16] By way of examples, let us mention some differences between **A**, **B** and **D** on the one hand and **C** on the other. In **A**, **B** and **D**, the description of the decimal places of an integer skips directly from the hundreds digit to the millions digit, while the corresponding sentence is complete in **C**: this is obviously the result of a homeoteleuton (**A**, f. 3a, **B**, p. 288, **C**, f. 3b, **D**, f. 4a). In **A**, **B** and **D**, the story of the Jew who solved an arithmetical problem related to Islamic law contains the sentence: *Kayfa yalīq bikum tuqallidūn yahūdiyy*$^{an}$ *malˁūn*$^{an}$ [How could that be appropriate for you to imitate an accursed Jew?], but the adjective *malˁūn* [accursed] does not exist in **C** (**A**, f. 17b, **B**, p. 308, **C**, f. 19b, **D**, f. 26b). In **A** and **D**, the statement of the first problem solved by double false position is hardly intelligible because its first words were accidentally omitted, whereas they are preserved in **C**: {*Mithāl min dhālik: rajul*$^{un}$ *tazawwaja bi-thalāth zawjāt wa-*} *dafaˁa li-kull wāḥida minhunna niṣf mahrihā*… [{An example of that: a man married three wives and} payed to each of them half of her dowry…] (**A**, f. 47a, **B**, lacking, **C**, f. 49a, **D**, f. 70a).

[17] By way of examples, let us mention some differences between **A** and **B** on the one hand and **C** and **D** on the other. In **C** and **D**, the first example of addition is 4563 + 5678 + 4567 + 3456 + 2345 = 20609, borrowed from Marco Aurel's book (Aurel 1552, 2b), but in **A** and **B**, it was modified to 4563 + 5679 + 4567 + 3456 + 2344 = 20609, probably because a copyist attempted to correct an earlier copyist's error (**C,** f. 4b, **D,** f. 5a, **A,** f. 3b, **B,** p. 289). In **C** and **D**, arithmetic progressions are defined as follows: *hiya allatī tatafāḍal bi-tafāḍul wāḥid mustawī*, which is the word-for-word translation of Aurel's wording: *los que se exceden por un ygual excesso* (Aurel 1552, 36a), but in **A** and **B**, the word *bi-tafāḍul* was omitted (**C,** f. 37a, **D,** f. 51b, **A**, f. 35a, **B,** p. 319). In **C** and **D**, the statement of a problem about Muslim soldiers looting Christian homes (translated and discussed below, see section 6) is complete, but in **A** and **B**, the end of the statement is lacking because of a homeoteleuton between two instances of the word *danānīr* (**C**, f. 42a, **D,** f. 59a, **A**, f. 40a, **B,** p. 327).

That is not all: major discrepancies exist between the four manuscripts. Manuscript **B** contains two considerable lacunae, of which the copyist was aware and which he attributed to the incompleteness of the manuscript he used. One of these lacunae concerns the final chapters: the text terminates prematurely and abruptly in the middle of a section devoted to the weighing plate method, *i.e.* the rule of false position. In **C** and **D**, this section is complete and is followed by a general explanation of the method of two weighing plates, *i.e.* the rule of double false position, and ten problems where this method is used; then the text ends with a very brief closure sentence.[18] The situation in **A** is somewhat in between: the text is essentially identical to that of **C** and **D** until the end of the first of the ten problems, but the diagram concluding this problem is missing as well as all nine other problems. Instead, and rather surprisingly, the text of **A** turns to a different series of small abstract problems solved by the method of two plates and then offers a long section consisting of various problems involving algebra, geometry and mathematical recreations of the Islamic tradition, which eventually ends very abruptly. All this additional material, contained only in **A**, apparently bears no relation whatsoever to Spanish sources: we believe it to be a late interpolation, not penned by Ibrāhīm al-Balīshtār himself. Although worthy of interest, it will not be considered it in the present article.[19]

Clearly, both **A** and **B** were copied from incomplete manuscripts: the copyist of **B** acknowledges this frankly, while the copyist of **A** clumsily attempted to fill in the lacunae. We believe that neither of them was copied from the other. But it could very well be the case that both were copied from one and the same manuscript **δ**, first **A** and later **B**, if we admit that some folios of **δ** had disappeared in the meantime. Especially, the vanishing of the last folio of **δ** would explain why **B** stops roughly two pages before the authentic part of **A**. Also, various errors in one exercise in multiplying fractions imply that **δ** and **D** were not copied from the same manuscript.

We provide below a tentative *stemma codicum* (Fig. 1), which is in some sense the minimal one that accounts for all discrepancies and textual variants between the four copies. Our conclusion is that **C** is by far the most complete and reliable witness of the original text. In a few cases, however, an error occurred in **C** and here the other manuscripts are likely to be closer to the archetype **Ω**.

[18] This sentence is: *Wa bi-llāh$^{i}$ al-tawfīq wa-ṣallā Allāh ˁalā sayyidinā Muḥammad wa-ˁalā ālihi wa-ṣaḥbihi wa-sallam.*
[19] However, let us indicate that some of the additional problems may have been drawn from Ibn Badr, from al-Fārisī (or maybe al-Karajī), from Ibn al-Bannāˀ's *Rafˁ al-ḥijāb* and *Maqālāt fī ˁilm al-ḥisāb* and from al-Ḥanbalī's commentary on Ibn al-Bannāˀ's *Talkhīṣ*.

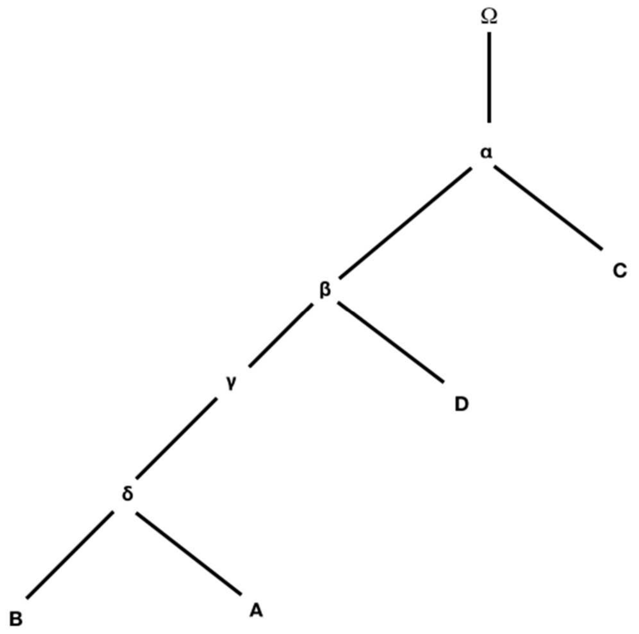


**Fig. 1** A tentative *stemma* of the four manuscripts preserved in Tunis

As displayed by this stemma, we need to postulate certain lost manuscripts of Ibrāhīm al-Balīshṭār's arithmetical treatise. Since the treatise was composed near to Algiers, we searched for copies in Algerian libraries, but so far have failed to locate any. The question then arises as to why four copies are kept in Tunis. Our hypothesis is that one, now lost, manuscript, say our stemma's **α,** might have been taken from Algiers to Tunis in the beginning of the eighteenth century and copied there several times. This is suggested to us by the following report by Ibn al-Muftī Ḥusayn bin Rajab, written around 1153/1740, about the decay of Algiers' great mosque library:

> <The librarian> al-Ḥāj Saʿīd was extremely negligent. In the days he delivered *fatwā*-s, he permitted to a number of people to take many books (…) Sīdī al-Ṭabbār al-Marūnī took a number of these volumes; after his death, his son carried them to Tunis and sold them (Ibn al-Muftī 2008, 100).[20]

During the revision phase of this paper, we were fortunate to locate a fifth copy, preserved in the Libyan Studies Centre in Tripoli. Because of the civil war in Libya, it is currently difficult to get hold of. We denote this copy by **E** and describe it shortly on the basis of this Centre's library catalogue (Sharīf 2006, 280) and a monograph about Libyan libraries (Sharīf-Ṭuwayr 1998, 62, 39-50):

**E**. The Libyan Centre for Manuscripts and Historical Studies (Tripoli), ms. 1096

[20] Other sources give al-Ṭāhir instead of al-Ṭabbār (Devoux 1866, 289), (Saʿadallāh 1998, I, 300). The ethnic name *al-Marūnī* might point to a Morisco or a renegade from Marrón (Cantabria), as was the Moroccan ambassador Aḥmad al-Ḥayṭi al-Marūnī.

This manuscript once belonged to the historian Aḥmad al-Nā'ib al-Anṣārī (1840-1918), the author of several books on the history of Tripoli. After his death, it was kept in the *Awqāf* [religious endowments] library of Tripoli and finally transferred in 1984 to the Libyan Studies Centre, then officially known as *Centre of the Libyans' Jihād against the Italian Aggression*. It has (only) 10 leaves, measures 14 cm by 20 cm and displays 17 lines per page written in Maghribī script.

**4. The sources of Ibrāhīm al-Balīshṭār's arithmetical treatise**

Identifying the Spanish book that Ibrāhīm al-Balīshṭār came across and eventually decided to translate in Arabic was a rather easy task, given the hint he gives about its author, whom he calls *Almān*: without a doubt, he meant the *Arithmetica Algebratica* authored by Marco Aurel, also known as Marco Alemán. This relatively little circulated book was printed in Valencia in 1552; we shall present it and its author in detail in section 5 of this article.

Later in his treatise, al-Balīshṭār speaks of "the problems mentioned in the books of the Christians", leaving with the impression that he may have resorted to other Spanish sources. Such is indeed the case. We discovered that he intertwined selected excerpts from Aurel's book with a number of problems found in another Spanish treatise, the very popular *Tractado subtilissimo d'Arismetica y de Geometria* by Juan de Ortega, first printed in 1512 and reprinted many times.[21] But for some unknown reason, he fails to make any mention of Ortega.

Al-Balīshṭār clearly expresses his admiration for some rules explained by *Almān* (Marco Aurel) and lacking in the books of the ancient Islamic scholars. One example is a set of algorithms for adding, subtracting and multiplying compound numbers, i.e. numbers containing units of different kinds (*diversas differencias*) like weights or amounts of money. He notes: "Concerning addition of multiple species, the masters (*shaykh*-s) that preceded us–may God have mercy on them–did not mention it and I have not seen anyone who mentions it except the author of the aforementioned book, who is the aforementioned priest".[22] Another example is the rule of alligation of metals, which we shall discuss in detail below. Occasionally, however, he mildly criticizes Marco Alemán and prefers methods of his own. For instance, for checking the result of a complicated computation of fractions like $\frac{1}{2}\frac{1}{6}\frac{1}{5}\frac{1}{7}\frac{1}{9}\frac{1}{8}\frac{1}{10}4320 = \frac{1}{70}$, he says that no method can be found in the books of the Ancients and that the one advocated by Alemán (Aurel 1552, 14a) would take a lot of time: for that reason, with God's help, he devised his own method.

[21] We shall refer to the Sevilla edition of 1552 (Ortega 1552), exactly contemporary to Marco Aurel's book. The differences with earlier or later editions seem insignificant for our purpose.
[22] For details on al-Balīshṭār's treatment of compound numbers, see (Hedfi 2019).

Al-Balīshṭār also refers to Arabic sources, thus making his treatise a fascinating cultural hybrid. Only two books, both written in central Maghreb during the fourteenth century, are explicitly mentioned:

(i) The commentary by Muḥammad b. Aḥmad al-Ghurbī (*ca.* 1349) on Ibn al-Bannāʾ's famous *Talkhīṣ*, a concise book on arithmetical operations.[23] Al-Balīshṭār mentions al-Ghurbī four times in his discussion of numerical progressions. He borrows some of his methods and occasionally quotes him almost word-for-word, but built different examples (see details in section 6 of this article).

(ii) The commentary by Saʿīd b. Muḥammad al-ʿUqbānī (d. 1408) on al-Ḥūfī's abridged book on the law of inheritance.[24] Al-Balīshṭār mentions it only once, saying that he came across al-ʿUqbānī's commentary and that he found there a support for his own understanding of fractions. Besides that, we conjecture that some inheritance problems stated by al-Balīshṭār were inspired by similar problems in al-ʿUqbānī's book.

In addition to that, al-Balīshṭār makes one passing reference to two famous earlier authors: he explains that "al-Ghurbī, Ibn al-Yāsamīn, Ibn al-Bannāʾ and others established only six [kinds of equations]". We recall that Ibn al-Yāsamīn (d. 1204) authored a *Poem on algebra*[25] which remained extremely popular in the sixteenth century, while the prolific Ibn al-Bannāʾ (d. 1321) was the most influential mathematician in medieval Maghreb. We also conjecture that two problems stated by al-Balīshṭār, concerning the shared purchase of a beast of burden, were inspired by similar problems in Ibn al-Bannāʾ's *Book of the foundations and preliminaries in algebra.*[26]

We cannot ascertain if al-Balīshṭār owned many Arabic mathematical books. We suspect that he did not have full or permanent access to the books he mentions. For instance, he rather vaguely says that he found a rule for multiplying any number by 9 "in some books on arithmetic by the Ancients" and that he invented a generalization for multiplying by 99…999. If he had read al-Ghurbī's treatise more closely, he would have found there an explicit statement of the general rule.

[23] Muḥammad b. Aḥmad b. Ḥasan al-Ghurbī, *Takhṣīṣ ūlī al-albāb fī sharḥ Talkhīṣ aʿmāl al-ḥisāb*. Five manuscript copies of this so far unedited text are known. For the purpose of this project, we consulted ms. 16320/1 in the Bibliothèque nationale de Tunisie and ms. D328 in the Bibliothèque nationale du royaume du Maroc. For short presentations, see (Harbili 2006) and (Lamrabet 2014, 186-187).

[24] Saʿīd b. Muḥammad b. Muḥammad al-ʿUqbānī al-Tilimsānī, *Sharḥ al-Mukhtaṣar fī al-farāʾiḍ li-l-Ḥūfī.* At least fifteen manuscript copies of this so far unedited text are known (Lamrabet 2014, 198). For the purpose of this project, we consulted ms. 571 in the Bibliothèque nationale de Tunisie and ms. Arabe 5312 in the Bibliothèque nationale de France. For a general presentation and a partial analysis in Arabic, see (Zarūqī 2000). For further details in French, see (Laabid 2006, 34-36, 101-110, 138-140, 190-197 and *passim*). A newly discovered copy is presented in (Aïssani *et al.*, 2016).

[25] Ibn al-Yāsamīn, *Urjūza fī al-jabr wa-l-muqābala.* The Arabic text together with English translation and analysis can be found in (Abdeljaouad 2005).

[26] Ibn al-Bannāʾ, *Kitāb al-uṣūl wa-l-muqaddimāt fī al-jabr wa-l-muqābala*. A critical edition of the Arabic text together with French translation and analysis can be found in (Djebbar 1990).

There are other aspects to hybridization. Al-Balīshṭār modified many of the problems he translated from the Spanish. On occasion, he adapted them to the Islamic law. In one case, he justified himself for not having done so: he argues that the rule of three with time (Aurel 1552, 23b) might be useful to Christian-born converts to Islam whose fathers had entered into a partnership before they died. He adjusted figures to the various systems of weights, measures and currencies in use in the Regency of Algiers. Moreover, some problems whose origin remains unknown to us have an unmistakably Islamic flavour: a typical example involves a man who pays to each of his three wives half of her dowry (see footnote 16 above). They might be an outcome of oral transmission, or simply of the author's inventiveness.

## 5. About Marco Aurel Alemán

This section is a digression providing background information about Marco Aurel. He is the author of two mathematical books, both written in Castilian and printed in Valencia. The first one, a short textbook aimed at merchants entitled *Tratado muy util y provechoso,* was published in 1541. The second, published in 1552, is the comprehensive and ambitious *Arithmetica Algebratica* that al-Balīshṭār undertook to translate into Arabic. In these books, Marco Aurel identified himself as a *natural Alemán*, i.e. a native German (Aurel 1552), and as *maestro de escuela*, i.e. a teacher (Aurel 1541). Further details were recently discovered in local archives. Marcho Alamany, as he was called in Valencian Catalan, taught writing and counting (*scriure e contar*) in the grammar classes of the university (*Estudi General*) of Valencia. He was never offered the chair of mathematics his skills deserved: on the 12th of June 1546, a medical doctor native of Spain was appointed instead of him; on the 30th of July, he waived the room he inhabited in the *Estudi General* and temporarily moved away from the city (Febrer Romaguera 2003, 74, 267, 452).[27]

We note that additional information can be deduced from Marco Aurel's coat of arms, as engraved in his books. The shield, consisting of what heraldic experts term a *cross pattée* and of four balls between the arms of the cross, is topped by a helmet supporting a *vol*, i.e. a pair of expanded and conjoined wings; moreover, the pattern of the shield is repeated inside each of the wings and the initials M.A. appear between them (Fig. 2). This coat of arms was used by various members of the Lieber–sometimes written Leiber–family, an old and prominent South German lineage from Augsburg, branches of which moved to Ulm in 1428, to Konstanz in 1439 and to Memmingen in 1517 (Leiber, 1976). Accordingly, there can be no doubt about Marco Aurel's real surname. We hope that this finding will facilitate future investigations about his life.

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[27] Primary source: Archivo Municipal de Valencia, *Querns de Provisions*, B-30, B-31.


**Fig. 2 Left:** Marco Aurel's coat of arms (Aurel 1541, 51b) **Right:** Sebastian Lieber's (1521-1594) coat of arms as displayed in a heraldic stained glass panel in a private choir chapel of Ulm Minster

We now give some details about Marco Aurel's first published book.[28] Its full title is: *Tratado muy util y provechoso, para toda manera de tratantes y personas aficcionadas al contar, de reglas breves de reducciones de monedas y otras reglas tanto breves quanto compendiosas* [Very useful and profitable treatise for all sorts of merchants and amateurs of reckoning about short rules on reduction of currencies and other rules, as brief as concise]. It was printed, rather poorly, in January 1541 by Francisco Díaz Romano, a native of Extremadure who ran a printing workshop in Valencia between 1531 and 1541, and is dedicated to George of Austria, an illegitimate son of Holy Roman Emperor Maximilian I, born in Ghent in 1505, who was appointed archbishop of Valencia in 1538 and actually lived there from January 1539 to July 1541 (Halkin 1936). The first part (fol. 1-33) consists in a series of fact sheets dealing with currency conversions. The second part (fol. 34-48), presumably relying on German sources, contains five unrelated sections: (1) a set of six rules allowing rough conversions from one another of the amounts per day, per month and per year of a rent or wage expressed with appropriate currencies; (2) an account of Boethius' cumbersome

[28] We are extremely grateful to Luis Puig from Valencia who provided us with a copy of this very rare book.

classification of proportions, a purely theoretical topic which the author included, he says, upon request of “certain gentlemen, friends and students of mine” (*algunos señores, amigos y discipulos mi*os);[29] (3) twelve linear problems of commercial arithmetic formulated in the language of proportions;[30] (4) six recreative problems (*Cuenta de plazer*) with solutions; (5) four unsolved partnership (*compañia*) problems, two of which are meant for beginners and the other two for advanced students.

Marco Aurel’s *magnum opus*, which would take him about a decade to write, was already forming in his mind by the time the small *Tratado* was printed: he announced his intention to write “another major book (…) about the rules of algebra, which in vulgar Castilian is understood by greater art or rule of the thing”.[31] It appeared in 1552 under the title: *Libro primero, de Arithmetica Algebratica, enel qual se contiene el arte Mercantiuol, con otras muchas reglas del arte menor, y la Regla del Algebra, vulgarmente llamada de Arte Mayor, o Regla de la cosa sin la qual no se podra entender el decimo de Euclides ni muchos otros primores, assi en Arithmetica como en Geometria (…) intitulado, Despertador de ingenios* [First book, about Algebraic Arithmetic, containing the Art of Commerce, and many other Rules of the Minor Art, and the rule of Algebra, vulgarly called Major Art, or Rule of the Thing, without which the tenth Book of Euclid cannot be understood, nor many beautiful things in Arithmetic as well as in Geometry (…) entitled the Awakening of the Ingenious]. It was dedicated to Bernardo Cimon, a local dignitary who was to become later first *jurado* of the citizens of Valencia, and printed in 1552 by Joan de Mey, a Flemish printer who had established in Valencia around 1535. Chapters 1 to 6 (fol. 1a-40a) deal with *art menor*, i.e. basic arithmetic. Chapters 7 to 12 (fol. 40a-68b) deal with roots and irrational numbers. Chapters 13 to 20 (fol. 68b-140a), deal with *art mayor*, i.e. algebra. The chapters on arithmetic have been rather neglected by historians of mathematics. By contrast, the sources of the chapters on algebra have been carefully studied by several authors.[32] Although never cited, the main source is beyond all doubt the celebrated German book known as the *Coß* (Rudolff 1525). Marco Aurel espoused Christoff Rudolff’s highly characteristic algebraic notations using Gothic script, followed his unusual classification of equations into eight types and reproduced many of his problems without any change–though he introduced two serious errors (Puig 2018). He also knew Luca Pacioli’s *Summa* (1494) and quoted it twice, but overall barely utilized it (Docampo Rey 2006, 46).

[29] This discussion seems to be borrowed from (Rudolff 1525, 49b-51b). It was repeated in (Aurel 1552, 14b-15b).

[30] As a sample, we reproduce the beginning of the second problem: *Un mercader quiere comprar por 600 ducados pebre, gengibre, canela y açafran, y quiere que las libras de gengibre con las libras de pebre esten en proporcion tripla, las libras del pebre y gengibre con las libras de canela en proporcion sesqui tercia, y las libras del açafran con todas las otras libras en proporcion dupla supertripartiens quartas…* (Aurel 1541, 41a). It is reminiscent of (Rudolff 1530, problem 255).

[31] In Spanish: *otro libro mayor (…) que tratara de las reglas de Algibra, que en vulgar castellano se entiende por arte mayor o regla de la cosa.*

[32] See (Meavilla Seguí 1993), (Romero-Vallhonesta 2018), (Romero-Vallhonesta and Massa-Esteve 2018), (Puig 2018).

In the prologue of his *Arithmetica Algebratica*, Marco Aurel explains that this book is intended to be the first of a set of three–hence the title *Libro primero.* In the second book, he planned to expose all geometrical proofs pertaining to the first, while the third one was intended to be devoted to practical geometry and aimed at craftsmen (*officiales mechanicos*). Although some authors have speculated about it (see the Epilogue below), there is no evidence whatsoever that the second and third books were actually written, let alone translated into Arabic. In fact, Marco Aurel completely vanishes from our sight after 1552. We were surprised to discover that the second part of his *Tratado muy util y provechoso* was reprinted in Saragossa in 1559 without his name: a publisher appended it to a new edition of a small accounting treatise, presenting it as an extract from the celebrated book by Juan de Ortega (de Icíar 1559).[33]

As we said, Ibrāhīm al-Balīshṭār referred to Almān as a priest–no less than five times throughout his treatise. However, sources provide no further evidence of Marco Aurel's clerical state.

### 6. Analysis of Ibrāhīm al-Balīshṭār's arithmetical treatise

Ibrāhīm al-Balīshṭār's treatise is untitled. It is organized as a non-hierarchical succession of unnumbered chapters (*abwāb*) and sections (*fuṣūl*). For the sake of clarity, we shall distinguish two parts. The first part outlines the basics of arithmetic: it contains some scattered mentions of Aurel's book–see section 4 above for examples–but almost nothing is directly translated from it. The second part is a compendium of problems on commercial transactions (*muʿāmalāt*), some of which–but not all–are translated or adapted from Aurel's or Ortega's book. Here is a rough table of the contents of the two parts according to **C**, the most complete and reliable of our manuscripts.

| | | |
|---|---|---|
| 1b | **Introduction** | |
| | | |
| 2b | ***<Part 1: The basics of arithmetic, for integers and compound numbers>*** | |
| 2b | **Chapter** | **Position of the dust figures and explanation of their places** |
| 2b | **Section** | **The places of numbers** |
| 4a | **Chapter** | **Addition** |
| 5b | **Chapter** | **Subtraction** |
| 7b | **Chapter** | **Multiplication** |
| 14a | **Chapter** | **Division** |
| 17b | **Chapter** | **Uses of fractions** |
| 20a | **Chapter** | **Addition of fractions** |
| 21a | **Chapter** | **Subtraction of fractions** |
| | | [thirteen problems involving addition or subtraction] |
| 22b | **Chapter** | **Multiplication of fractions** |
| | | [six problems] |
| 23b | **Chapter** | **Division of fractions** |

[33] This appendix was commented in (Ausejo 2015) without calling into question its attribution to Ortega.

| | | |
|---|---|---|
| | | [seven problems] |
| 22a | **Section** | **<Fractions of fractions>** |
| | | [five problems] |
| | | |
| 25b | ***<Part 2: Problems on commercial transactions>*** | |
| 25b | **Chapter** | **How to work with the <rule of> three proportional numbers** |
| 25b | | [four problems of unknown provenance] |
| 26b | | **How to work out proportional share (*nisbat al-muḥāṣāt*)** |
| | | [five problems of unknown provenance] |
| 28a | | **Exercises on the rule of three proportional numbers, so that you can observe deep and delicate problems** |
| | | [eight problems: the first two and the last one are of unknown provenance, the five remaining are taken from (Aurel 1552, 22b-23b)] |
| 29b | | **Problems mentioned in the books of Christians** |
| | | [six problems: the first one is taken from (Aurel 1552, 23b-24a), the second and the third ones are variations of the first, the last three are taken from (Aurel 1552, 24b-25a)] |
| 30b | **Section** | **The rule of five proportional numbers** |
| | | [six problems: the first one seems to be inspired by (Ortega 1552, 100b), the last five are taken from (Aurel 1552, 26b-29a)] |
| 32b | **Section** | **Other <barter-type> transactions** |
| | | [five problems: the first one is taken from (Aurel 1552, 29b), the last four are taken from (Ortega 1552, 157b-159b)] |
| 34a | **Chapter** | **On the rule of alligation** |
| | | [seven problems taken from or inspired by (Aurel 1552) and (Ortega 1552), see details below] |
| 35a | **Section** | **Something from the rules of ratios and excesses <of numerical progressions>, from a concise point of view** |
| 36b | **Section** | **Adding numbers exceeding each other by two different excesses <alternately>** |
| 38a | **Section** | **<Adding> numbers exceeding each other <by excesses in arithmetic progression>** |
| 38b | **Section** | **Addition over a geometric <progression>** |
| | | [eight problems on numerical progressions inspired by al-Ghurbī, see details below] |
| 42b | **Section** | **Problems that can be answered using proportionality and the rule of one plate <i.e. the method of simple false position>, and after that, problems that cannot be answered save by the rule of two plates, or by algebra** |
| | | [35 problems solved by simple false position: at least four of them are taken from (Ortega 1552, 198b-201b), some other are similar to problems found in books by Ibn al-Bannāʾ and al-ʿUqbānī, and some may very well have been devised by the author himself] |
| 48b | | **How to work with two plates <i.e. the method of double false position>** |
| | | [nine problems] |
| 53b | | [the text end abruptly at this point] |

Just after the method of double false position, most Arabic arithmetical books would include a chapter on algebra. This is notably true for Ibn al-Bannāʾ's *Talkhīṣ* (concise book) and its numerous commentaries, including those by Ibn al-Bannāʾ himself, by al-Hawārī al-Miṣrātī, by al-Ghurbī, by Ibn al-Haydūr, by al-Qalaṣādī and many others (Abdeljaouad and Hedfi 2018, 89-102). There is no such chapter in the preserved manuscripts of al-Balīshṭār's treatise; however, it seems obvious that he wrote one or intended to do so. Indeed, in the text as we have it, there are several mentions of algebra (*al-jabr wa-l-muqābala*), also called the great art (*al-ṣanʿ al-akbar*) or the rule of the thing (*qāʿidat al-shayʾ*), and a specific chapter (*bāb*) on this topic is announced explicitly. Moreover, algebraic techniques already appear in a section about numerical progressions, where four problems are solved through quadratic equations (see below); al-Balīshṭār even draws attention to the fact that these equations are classified differently in Arabic sources and in Almān's book. The lack of conclusions and the abrupt end reinforce our impression that his treatise is a mutilated or uncompleted work.

**The chapter on the rule of alligation.** This chapter is particularly worthy of interest since al-Balīshṭār considers the arithmetical rule of alligation (*qāʿidat al-amzija*), i.e. alloying of metals, an unprecedented achievement of the Christians: "I have not seen anyone among the ancients (*al-qudamāʾ*) mentioning it, but it is a nice rule". This rule, which appears in many treatises written in Europe during the fourteenth, fifteenth and sixteenth centuries, goes back to Leonardo of Pisa, known as Fibonacci: in chapter eleven of the final form of his *Liber Abaci*, completed in 1228, no less than seven kinds (*differentiæ*) of alligation problems are discussed, with many examples; at one point, Fibonacci refers to his now lost *Liber Minoris Guise* (Caianiello 2011). Actually, earlier Arabic mathematicians had occasionally dealt with alligation problems. In the eleventh century, the Baghdad mathematician al-Karajī had a few of them in his *al-Fakhrī* (Karajī 1986a, 180, 190) and *al-Kāfī* (Karajī 1986b, 184-187), which he solved solely by algebra. Also in Baghdad, a series of alligation problems appeared in a compilation in applied mathematics from the twelfth century entitled *al-Ḥawī* (Šaqqāq 2008, 140-143), in which they were solved by an arithmetical rule slightly different from Fibonacci's. The Arabic tradition in statics, as synthetized in the twelfth century by al-Khāzinī's in his book *Mizān al-ḥikma* [The Scale of Wisdom], was interested in determining the composition of a given alloy. Late arithmetical treatises occasionally contained problems where unspecified ingredients are mixed: there is one such problem in the book *al-Tamḥīṣ* by the Moroccan scholar Ibn Haydūr (d. 1413), solved by the rule of double false position.[34]

Al-Balīshṭār's own treatment of alligation consists of seven problems. It is based on both Aurel's very short account of the *Regla de ligar oro y plata,* which consists in two problems only (Aurel 1552, 30b-31a), and on Ortega's lengthy sections entitled *Dela fineza dela plata* and *Dela fineza del*

[34] ʿAlī b. ʿAbdallāh b. Muḥammad Ibn Haydūr al-Tādilī, *al-Tamḥīṣ fī sharḥ al-Talkhīṣ* [Scrutiny in the commentary on Ibn al-Bannāʾ's *Talkhīṣ* (concise book)]. Eight manuscript copies of this text are known by now. For the purpose of this project, we consulted ms. 12467 in the Bibliothèque nationale de Tunisie (f. 194b). For short presentations in French, see (Abdeljaouad and Hedfi 2018, 101-102) and (Lamrabet 2014, 199-202). For a detailed presentation in Arabic with selected and commented excerpts, see (Jābirī 2016).

*oro* (Ortega 1552, 168a-183a). Let us go into more detail and denote al-Balīshṭār's problems by R1,…, R7.

We will not say much about problems R3, R4, R5. They relate to the situation traditionally known in English as *alligation medial*: quantities of several metals and the fineness of each being given, to find the fineness of the alloy. In modern notations, it amounts to computing the weighted mean $a = \frac{\sum_{i=1}^{n} a_i x_i}{\sum_{i=1}^{n} x_i}$. In R5 however, where four kinds of silver are considered, the fineness of the fourth kind is sought for while the fineness of the alloy is given. These three problems are taken from (Ortega 1552, 171b-173b). The only changes consist in some conversions of units.

Problems R1, R2, R6 and R7 deal with the more complex task of *alligation alternate*: the fineness of each of several metals being given, to find what quantities of them should be melted to get a given quantity of alloy of a given fineness. In algebraic terms, its amounts to finding a positive rational solution of a system of two linear equations of the form $\sum_{i=1}^{n} a_i x_i = a \sum_{i=1}^{n} x_i$ and $\sum_{i=1}^{n} x_i = b$. Such a problem has at most one solution for two kinds of metals, but it is underdetermined for three kinds or more: in the latter case, practitioners were only interested in finding one particular solution. In what follows, we shall use algebraic notations for convenience, but it is essential to note that the methods advocated by al-Balīshṭār do *not at all* resort to algebra.

Problem R1 is most faithfully translated from Aurel's book (Aurel 1552, 30a-b):

> [R1] **Problem.** A goldsmith has two kinds of gold: one of them is of 15 carats of fineness, and the other of 23 carats. He wants to make out of them gold of fineness 20 carats. How much does he take from each of the two kinds so that the whole thing mixed together be of 20 carats of fineness?[35]
>
> Draw for this purpose a weighing scale, like this. Then you observe by how much one of the two kinds exceeds 20.[36] You find it to be 3 carats. Lay it above, as you see. Then you observe by how much 20 exceeds 15. You find 5. Then you say that for any three parts of gold 15, he ensures to put five parts of gold 23. If mixed according to this proportion, the alloy will be of 20 carats of fineness as prescribed.
>
> The proof of it is: multiply 5 by 23. There results 115; save it. Then multiply 3 by 15, the fineness of its kind: there results 45. Add it to the saved <number>; this is 160. Divide by 8, the sum of the parts taken from <each> kind. The result of the division is 20 and this is the value asked for.

[35] We give the Arabic and Spanish versions as samples for comparison. In Arabic: *Sāʾiʿ lahu jinsān min dhahab : aḥdahumā min 15 qirātan min ʿuluww, wa-l-ākhar min 23 qirāṭ$^{an}$ : wa-arada an-yaʿmal minhumā dhahab$^{an}$ yakūn ʿuluwwuhu 20 qirāṭ$^{an}$. Fakam yaʾkhudh min kull wāhid min al-jinsayn, bi-ḥaythu annahu idhā muzija al-jamīʿ kāna ʿuluwwuhu 20 qirāṭ$^{an}$ ?* In Spanish: *Vn platero tiene dos suertes de oro : la vna es de 15 quilates, y la otra de 23 quilates de ley : quiere hazer oro de ley de 20 quilates. Demando, quanto tomara de cada suerte, para que juntado en vn crisol venga a ser de 20 quilates de ley.*

[36] All manuscripts give here "exceeds the other", but this is clearly a mistake.

In algebraic terms, it amounts to solving the equation $15x_1 + 23x_2 = 20(x_1 + x_2)$, hence $3x_2 = 5x_1$. The value of $x_1 + x_2$ is not specified. The solution resorts to a specific procedure, which is in fact Fibonacci's *Differentia sexta*, guided by a diagram which does not bear any name in Aurel's book. Interestingly, al-Balīshtār slightly modified this diagram and called it a *mizān* (weighing scale), which suggests that he had built his own understanding of the method in terms of weights (Fig. 3).

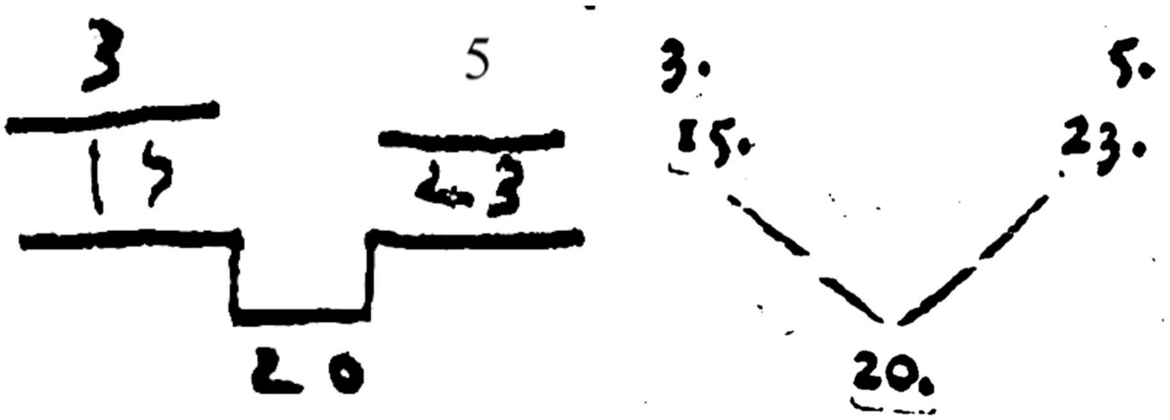


**Fig. 3 Left:** Al-Balīshṭār's scale for alligation problem R1, with restoration of a figure forgotten by the copyist (ms. BN Tunis 16497/1, fol. 34b) **Right:** Aurel's diagram for the same problem (Aurel 1552, 30a)

The proof (*prueva*) offered by Aurel is to be understood in the sense of a test, trial, or experiment: he checked that the procedure gives the correct result in the example considered, but did not attempt to justify it in the general case. Al-Balīshṭār reproduced this proof in an abridged form and called it an *ikhtibār*: this Arabic word unambiguously refers to an experiment, not to a logical inference. At this point, he deviated from Aurel's approach. Whereas the latter did not offer any example with more than two kinds of gold, al-Balīshtār considered a generalization to multiple kinds:

> If he <the goldsmith> wants to take from multiple kinds <of gold>, then the practice in this matter is to lay a weighing scale in the manner of the one placed <previously>. Then, lay the kinds that are greater than the <fineness> required on the right side of the scale over the line, and those that are less than it on the left side of it. Then place on each line a separation, as you see in the scale before. Then lay the required value under the beam (*qubba*) of the scale.

Problem R2 is an example involving five kinds of gold:

> [R2] **An example of that.** If we want to take from gold of strength (*quwwa*) 24 carats, from gold 20, from gold 16, from gold 15 and from gold 14 and that the whole thing be mixed into gold of strength 18 carats. You say, after disposing the scale according to the aforementioned description (…)

It amounts to solving $24x_1 + 20x_2 + 16x_3 + 15x_4 + 14x_5 = 18(x_1 + x_2 + x_3 + x_4 + x_5)$, hence $6x_1 + 2x_2 = 2x_3 + 3x_4 + 4x_5$. The value of $x_1 + x_2 + x_3 + x_4 + x_5$ is unspecified. As only one particular solution is sought for, al-Balīshṭār implicitly imposes $x_1 = x_2$ and $x_3 = x_4 = x_5$. For this purpose, he arranges a *mizān* as in problem R1, computes $24 - 18 = 6$, $20 - 18 = 2$, $18 - 16 = 2$, $18 - 15 = 3$ and $18 - 14 = 4$, lays these values out in columns as follows and adds up each column (Fig. 4). He obtains the solution $x_1 = x_2 = 9$ and $x_3 = x_4 = x_5 = 8$.

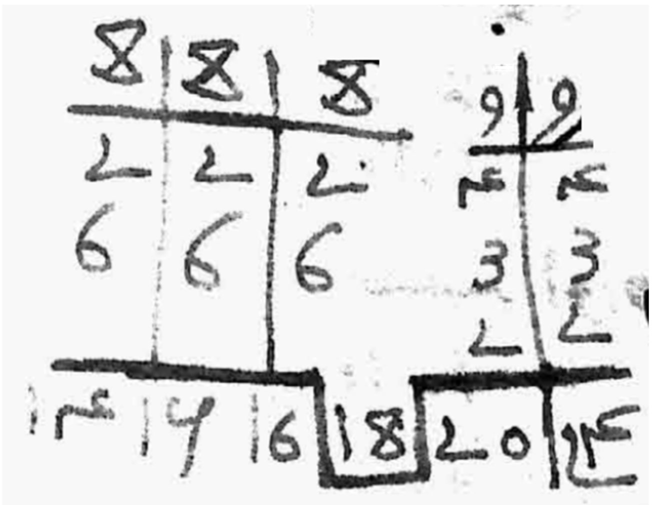

**Fig. 4** Al-Balīshṭār's scale for alligation problem R2 (ms. BN Tunis 16450/1, fol. 32a)

This problem may have been inspired by Ortega's treatise, where a similar question is raised and solved in an essentially similar way (Ortega 1552, 182a-183a). It involves six kinds of gold and amounts to $15x_1 + 17x_2 + 18x_3 + 20x_4 + 21x_5 + 23x_6 = 19(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)$. As this equation leads to $x_3 + 2x_4 + 4x_5 = 4x_1 + 2x_2 + x_3$, it happens here that a particular solution is obtained when all six kinds are mixed in identical proportions. Since Ortega adds the further requirement $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 50$, he obtains $x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = 8\frac{1}{3}$. Why did al-Balīshtār modify Ortega's problem? Our interpretation is that he wanted, for didactic purposes, to avoid that potentially misleading situation.

Problem R6 involves two plates of silver. It boils down to the system of equations $16x_1 + 8x_2 = 10(x_1 + x_2)$ and $x_1 + x_2 = 18$, whose unique solution is $(x_1, x_2) = (4\frac{1}{2}, 13\frac{1}{2})$. This problem is taken from (Ortega 1552, 173b), with no change apart from a simple conversion of units.[37] The accompanying diagram resembles that by Ortega, but is not a *mizān* as above (Fig. 5).

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[37] The rules applied are: *1 ducado → 2 dīnār ziyānī*, *1 marco → 2 raṭl*. The *dīnār ziyānī* was a fine gold coin, ordinarily minted in Tlemcen and also in Oran, Ténès, Algiers and Béjaïa. After the fall of the Zianite kings in 1556, the Turks continued to mint such coins for some time.

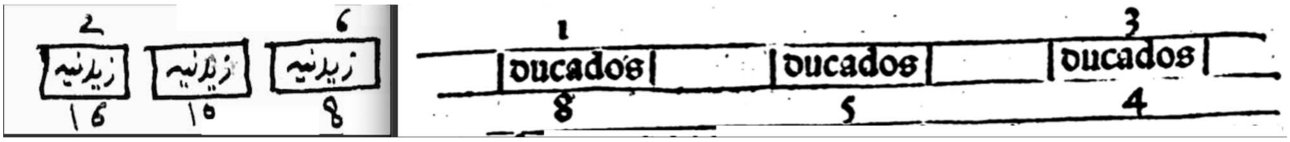

**Fig. 5 Left:** Al-Balīshṭār's diagram for alligation problem R6 (ms. BN Tunis 16497/1, fol. 36a)
**Right:** Ortega's diagram for the same problem, with conversion of units (Ortega 1552, 173b)

Problem R7 is about a mixture of gold and silver and amounts to the determinate system

$$\begin{cases} 16x_1 + 3x_2 = \frac{1028}{100}(x_1 + x_2) \\ x_1 + x_2 = 100 \end{cases}.$$

It is solved by the technique of the *mizān* as in R1, except that all numbers are multiplied beforehand by 25 to avoid fractions. The solution is $(x_1, x_2) = (44, 56)$. This problem and its solution are similar to Aurel's second problem (Aurel 1552, 30b-31a), which is also about gold and silver and leads to:

$$\begin{cases} 64x_1 + 24x_2 = \frac{738}{30\frac{1}{4}}(x_1 + x_2) \\ x_1 + x_2 = 30\frac{3}{4} \end{cases}.$$

But Aurel's problem involves complicated fractions and its solution is $(x_1, x_2) = (21\frac{44}{76}, 9\frac{13}{76})$. It seems that al-Balīshtār modified it in order to avoid these fractions, for didactic purposes. On the other hand, he made the problem slightly longer, since the numbers searched for in his version are not the quantities $x_1$ and $x_2$ of gold and silver, but the sharing of a profit of 514 dinars:

$$g_1 = \frac{16x_1}{16x_1+3x_2} \times 514 = 448; \quad g_2 = \frac{3x_2}{16x_1+3x_2} \times 514 = 66.$$

**The sections on numerical progressions and their sums.** These highly interesting sections come immediately after the chapter on the rule of alligation. At the beginning, al-Balīshṭār closely follows Marco Aurel's treatment of the topic in the chapter called *Trata de progressiones, y de reglas generales para summarlas* (Aurel 1552, 35b-40a). Thereafter, he suggests that European arithmeticians were outperformed by their Muslim counterparts and formulates eight problems inspired by his reading of al-Ghurbī's commentary, which he solves using quadratic algebra.

Let us go into a bit more detail. Following Aurel, al-Balīshṭār recalls that the scientists of the past spoke extensively about the sum of the integers, the odd numbers and the even numbers, and claims that these cases can be subsumed under one unique rule. He also reproduces, not quite faithfully, Aurel's threefold classification of the progressions. Then he gives the rule for computing the sum of an arithmetic progression: in modern notations, this sum is $S = \frac{n}{2}(a + A)$ where $n$ is the number of terms, $a$ is the least term and $A$ is the greatest one. He reproduces all of Aurel's illustrating examples. Leaving out some material (the wheat and chessboard problem, the rules for computing the sum of consecutive squares or cubes), he resumes his translation in stating, without proof, two generalizations of the rule for an arithmetic progression: one for progressions like (7, 10, 15, 18, 23, 26) such that the differences between two consecutive terms alternate between two values, and one for progressions like (5, 12, 26, 47, 75) such that these differences are in arithmetic progression. In the latter case, the proposed method can be encoded in the following formula: $S = (\frac{n-2}{3} + 1)(A - a) + na$. The rule for computing the sum of a geometric progression is also translated from Aurel and is illustrated by the same example: $\sum_{n=0}^{5} 2.3^n = 728$. Its formulation may be confusing for a modern reader since it adopts the point of view that the common ratio of the progression is a proportion, *i.e.* a pair of numbers, and not a single number. But al-Balīshṭār proudly offers a "marvellous shortening" (*ikhtiṣār badīʿ*) that he claims to be of his own invention, which amounts to the modern formulation of the rule. Then, referring to al-Ghurbī, he considers the situation where the sum is given and other parameters are unknown. He states and solves three small problems he composed specially for students:

> [S1] Boxes, in proportion of one fifth, in the first of them is 5 and the sum is 3905. [What is the greatest?]

> [S2] Boxes, in proportion of one fifth, the greatest is 3125 and the sum is 3905. What is the least?

> [S3] Boxes, in the first of them is 5, the greatest is 3125 and the sum is 3905. How much is the ratio?

Al-Balīshṭār points out Aurel's book's shortcomings in regards to such problems and writes:

> As for the priest Almān, the only rule he established is the one just presented, the rule for determining the sum, nothing else.

Reverting to arithmetic progressions, he takes a similar approach. He writes:

> God willing, we shall present examples of the arithmetic ratio such as those mentioned by al-Ghurbī for the purpose of his commentary to the *Talkhīṣ* of Ibn al-Bannāʾ–may God have mercy on both of them. They are: if the least extreme (*al-ṭarf al-aṣghar*) is unknown, or the greatest (*al-akbar*), or the difference (*al-tafāḍul*), or the number of the numbers (*ʿiddat al-aʿdād*), or the sum (*al-jumla*). If the

least (or the greatest) extreme and the difference are unknown, divide the sum through half of the number of numbers: there results the sum of the two extremes; then, if the greatest is subtracted from it, the least remains, and if the least is subtracted, the greatest remains. […] As for the least and the sum, multiply the number of numbers minus one by the difference: the result is the excess of the greatest overs the least. Subtract it <from the greatest>: the least remains. The, add them and multiply their sum by half the number of numbers: the sum results. You [can] also do the job if the least[38] <number> and the number of numbers are unknown: it is possible to determine them, but only through the fifth kind <of equation> in algebra. For Almān, it is the sixth kind, because he established eight kinds, while al-Ghurbī, Ibn al-Yāsamīn, Ibn al-Bannāʾ and others established only six of them.

He proposes five original problems pertinent to al-Ghurbī's rules. Their statements are:

[S4] If you are told: ten boxes (*buyūt*), the greatest of them is 21 and the sum is 120. How much is the least and how much is the difference?

[S5] If you are told: numbers, their sum is 38, the greatest extreme is 14 and the difference is 3. How much is the number of the numbers and how much is the least extreme?

[S6] If you are told: numbers, their sum is 109536, the greatest extreme is 661 and the difference is 2. How much are the number of the numbers and the least extreme?

[S7] If you are told: numbers, they differ by 2, the greatest is 14699 and the sum is 54022451. What is the least [number] and what is the number of numbers?

[S8] Suppose that the fleet of Algiers invaded one of the cities of the Christians–may God destroy them. They took it and looted all of its houses. They found in the first house a certain number of dinars, known as escudos. In the second house, they found 100 dinars of silver more than what was in the first house. Then they found in the third house 100 dinars of silver more than what they had found in the second house. The houses of the city differed like this by 100 dinars until the last house. In the last house, they found 636970 dinars. Then they added everything; the sum was like this: 2028972400, all of them dinars of the said species. How many houses did they find in the city, and how many dinars did they find in the first house?

Problems S5 to S8 are all of the same type. As announced, they are solved by establishing a quadratic equation of the fifth kind whose unknown quantity is the number of numbers. Indeed, in modern terms, the relations $S = \frac{x}{2}(a + A)$ and $A = a + (x - 1)r$ imply $\frac{r}{2}x^2 + S = (A + \frac{r}{2})x$. Recall that Arabic algebra considered only positive numbers and that the fifth kind of equation was

[38] In all manuscripts: greatest. This is an error that might be the result of a homeoteleuton.

what we would write as $ax^2 + c = bx$, with $a > 0, b > 0, c > 0$. Al-Ghurbī worked up the simple example where $r = 2$, $A = 10$ and $S = 30$, thus obtaining $11x - x^2 = 30$. He solved this equation and obtained $x = 5$, hence $a = 2$, ignoring the solution $x = 6$ which would imply an inacceptable $a = 0$. Al-Balīshṭār replaces that example by the slightly more complicated problem S5, which leads to the equation $38 + \frac{3}{2}x^2 = \frac{31}{2}x$. He obtains $x = 4$ and ignores the inacceptable solution $x = \frac{19}{3}$. In fact, he is aware that algebra can be avoided by using a simpler "working backwards" method and does point this out, but he warns that it becomes cumbersome when the sum is a large number. Problem S6, which he singles out as especially difficult, is intended to illustrate this point: it leads to the equation $662x - x^2 = 109536$. The similar problem S7 is designed to give to the student an opportunity to show his excellence: it leads to the equation $14700x - x^2 = 54022451$. In these two cases, al-Balīshṭār does not mention that the quadratic equation has two integer solutions (326 and 336 for S6, 7343 and 7357 for S7): he only computes the least one. In fact, in both cases, the other solution is not acceptable since it does not fulfill the condition of having $a > 0$, which is equivalent to $x < 2S/A$. Finally, problem S8 is the most impressive because of the very large numbers involved. It leads to the equation $637020x - 50x^2 = 2028972400$ whose only integer solution is 6370. Al-Balīshtār concludes with the following comment, displaying–or alleging–an intense level of mutual hatred between Muslims and Christians:

> I composed this problem in this form in retaliation against the unbelievers–may God destroy them–because I found in their books dealing with this art many examples where they hope for the capture of Algiers the well-guarded. But God annihilated them and He lowered their flags. May He maintain Algiers in Islam, by the intercession (*jāh*) of the Prophet–Peace be upon him. God willing, we shall mention examples of this in the chapter on algebra and shall retaliate for them against unbelievers.

As we have already mentioned, the advertised chapter on algebra was either not written or not preserved in the manuscripts we have. The existence of a section involving complicated algebraic equations before exposition of the basic rules of algebra may seem awkward; however, exactly the same phenomenon can be observed in al-Ghurbī's commentary. Another remark is that al-Balīshṭār follows al-Ghurbī in occasionally using the traditional Maghrebian algebraic symbols.[39] E.g., in problem S6, the algebraic expression $661-(2x-2)$ is displayed as follows (Fig. 6):

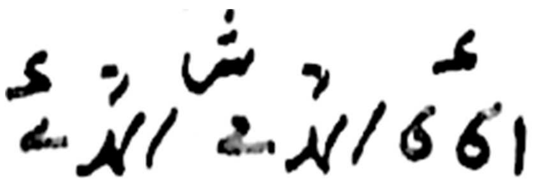

**Fig. 6** Maghrebian algebraic symbols (ms. BN Tunis 16497/1, fol. 41a)

The symbolism is to be deciphered as follows: from right to left, we find successively the number 661 topped with the first letter ع of the word *ʿadad* (number), the word الّا (*illā*) denoting a subtraction, the number 2 topped with the first letter ش of the word *shayʾ* (thing); another instance of الّا, which is by convention at a lower arithmetical level than the previous one, and the number 2 topped with the first letter ع of the word *ʿadad* (number).

Finally, we wondered about the mathematical problems planning the capture of Algiers mentioned by al-Balīshṭār in problem S8. Although he claims that the Christians have many of them in their books of arithmetic, we could unearth only one such example. It appears in the section about the first type of equation in Marco Aurel's treatise (Aurel 1552, 96b) and begins with these words: "The King our Lord recruits people to <go to war in> Algiers, among whom there are Spaniards, Germans and Italians."[40] This military 'dressing-up' of a system of linear equations with three unknowns is probably an allusion to Charles V's failed expedition to Algiers in 1541, for which it was planned to recruit 7000 Spaniards, 6000 Germans and 5000 Italians (Nordman 2011, 139).

## 7. Epilogue: a historiographical conundrum

During our research, we were surprised to discover that several biographical sketches of Marco Aurel available on the web vaguely mention the fact that his book was translated into Arabic,

[39] About these symbols, see: (Abdeljaouad 2002), (Oaks 2012), (Lamrabet 2014, 349-353).
[40] In Spanish: *El Rey nuestro señor haze gente para Alger, entre la qual ay, Españoles, Alemanes, & Italianos.*

although nobody was apparently aware of the manuscripts we discovered in Tunis.[41] After further investigation, it turned out that the original source for this piece of information is a short note written several decades ago by Juan Vernet. Let us translate the whole of this note:

> In the prologue of the work *Glory and Benefit for those who fight with cannons* of Ibrāhīm b. Aḥmad Ġanim Arribas, whose Arabic version is due to Bejarano, it is deduced that "a book by Alemán, the most important that exists regarding arithmetic, algebra and geometry", was also translated into Arabic. It seems to me that we encounter here a quote of exceptional importance, since it must refer to the work of Marco Aurel Alemán *Libro primero de aritmética algebrática* (Valencia, 1552) that introduced in Spain the algebraic notation of the German authors and seems to be inspired, directly, by that of Rudolff. Furthermore, if the Arabic text that preserves for us these pieces of information is exact, it can be believed that Aurel did publish the last two parts of his work, of which no record seems to remain in the Spanish libraries (Vernet 1974, 645).[42]

There is a problem with Vernet's account. The treatise on artillery by Ibrāhīm b. Aḥmad Ghānim, also known as Rivas or al-Ribāsh, is well-known.[43] It is a compilation from Spanish sources, written in Spanish in 1631 by a Morisco from Nigüelas (Andalusia) who lived in Tunis. As Vernet says, it was later translated into Arabic by another Morisco, Aḥmad al-Ḥajarī, alias Bejarano.[44] Only the Arabic version survives, in at least ten manuscripts. We inspected some of them and found absolutely nothing related to the book by Alemán. Clearly, Vernet's claim was not the result of a direct examination. So we went to the reference cited by him in his note: an article in Arabic by the Moroccan scholar Muḥammad al-Mannūnī, published ten years earlier in a Spanish journal. It turned out that the translation of "a book by Alemán" is, indeed, mentioned in this article, but that Vernet had misunderstood the relevant bibliographical footnote, in which the reader was, somewhat ambiguously, referred to "the previous source" (Mannūnī 1964, 342-343). These words did not point to Ibrāhīm Ghānim's treatise of artillery, as Vernet assumed, but to a book by ʿUthmān al-Kaʿʿāk, printed in Cairo in 1958. So we set about searching for this book.

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[41] E.g. on the Internet site of the Spanish Royal Academy of History: *Tambien (…) en Marruecos se tradujo al árabe, no sólo la primera parte, la única conocida en castellano, sino también la segunda y la tercera* (http://dbe.rah.es/biografias/19071/marco-aurel, consulted on 5 November 2019).

[42] In Spanish: *En el prólongo [*sic*] de la obra* De la fuerza [*sic*, for Arabic *'izz*] y de la utilidad para quienes combaten con cañones *de Ibrāhīm b. Aḥmad Ġanim Arribas, cuya versión árabe se debe a Bejarano, se deduce que también se vertió al árabe "un libro de Alemán que es el más importante que hay en cuanto a aritmética, álgebra y geometría". Me parece que aquí nos encontramos con una cita de excepcional importancia, pues debe referirse a la obra de Marco Aurel Alemán* Libro primero de aritmética algebrática *(Valencia, 1552) que introdujo en España la notación algébrica de los autores alemanes y parece inspirarse, directamente, en la de Rudolf [*sic*]. Además, si el texto árabe que nos conserva estas noticias es exacto, puede creerse que Aurel publicó las dos últimas partes de su obra, de las cuales no parece quedar constancia en las bibliotecas españolas.*

[43] The exact title is: *al-ʿIzz wa-l-Manāfiʿ li-l-mujāhidīn fī sabīl Allāh bi-ālāt al-ḥurūb wa-l-madāfiʿ* [Glory and Benefits for those who fight on the way of God with war machines and artillery].

[44] Al-Ḥajarī is also known for his translation out of Spanish into Arabic of Zacuto's *Almanach Perpetuum*. In addition, he undertook to translate the French edition of the Mercator-Hondius *Atlas* and, in partnership with a priest captive in Morocco, the Latin edition of Hues's *Tractatus de Globis*: these translations are not extant today (Ageron 2019, 95-99).

ʿUthmān al-Kaʿʿāk (1903-1976), or Othman Kaak in the spelling in use in Tunisia, was a Tunisian scholar who was the head of the Department of Arabic books in the Bibliothèque publique de Tunis and was promoted in 1956 to General Curator of the same library, since then referred to as Bibliothèque nationale de Tunisie. He held this position until 1965. The small book in Arabic he published in 1958–rather a project of book, he admits in the foreword–is entitled: *Lectures on centres of culture in the Maghreb from the sixteenth century to the nineteenth century*. It was not widely circulated, and we are grateful to the University Library of Aix-en-Provence for kindly sending us a copy of it. In this book, ʿUthmān al-Kaʿʿāk gave some details about Ibrāhīm Ghānim and added, without any further precision, the following unprecedented and stunning claim:

> He [Ibrāhīm Ghānim] also translated from Spanish the book by 'Almān', which is the greatest book on arithmetic, algebra and geometry (Kaʿʿāk 1958, 98).

These few words clearly imply his awareness of the existence of the mathematical treatise under study in the present article. He may of course have glanced at one of the four manuscripts known to us, specifically **D** which was kept in the library of which he was in charge. It is also possible that a document of a different kind was available to him. Indeed, his account does not match with any of the four Tunis manuscripts: only a very quick look at any of them is necessary to ascertain that the treatise contains little about algebra, nothing on geometry and that the translator is not Ibrāhīm Ghānim al-Ribāsh–who, anyway, was ignorant of Arabic–but Ibrāhīm al-Balīshṭār. We tried to browse through ʿUthmān al-Kaʿʿāk's archives at the Bibliothèque nationale de Tunisie, but could not find anything relevant to this issue. His elusive and partly flawed claim therefore remains a conundrum. As explained before, it made its way into the historical scholarship down to the present day.

Although his misreading of al-Mannūnī's book added further confusion, Juan Vernet was right in one respect: his conjecture that Alemán is Marco Aurel was perfectly correct. But his often echoed speculation that the two volumes of geometry that Aurel intended to publish could have been preserved in Arabic while they are unknown in Castilian was built on sand and was rather far-fetched.

## Conclusion

Recent research has revealed that intellectual life among Aragonese and Valencian Moriscos of the sixteenth century was much more active than what was generally assumed. Ibrāhīm al-Balīshṭār's treatise highlights an aspect of it that faded into oblivion with time: the cultivation of mathematics. Proud of his Muslim identity, but also of his Spanish origin, al-Balīshṭār tried to reunify two scientific traditions sharing common roots. Even though it appears to be unfinished or mutilated, the pedagogical treatise on arithmetic that he composed in Cherchell is a rich and elaborate personal work, blending Spanish and Arabic sources, freely adapted and commented on. The five extant copies witness a significant and lasting circulation in Northern Africa.

Although al-Balīshṭār presented his work as a translation of Alemán's (i.e. Marco Aurel's) book, it turned out from our research that he translated a fairly limited amount of the material included in this book. Conversely, a small part only of his treatise was borrowed from Aurel's book: besides various Arabic sources, he used other European sources, notably the book by the Dominican priest Juan de Ortega. A series of intriguing questions then arises. Why did al-Balīshṭār constantly refer to the mathematician "Almān", expressing deep respect and admiration for him despite his hatred of Christians? Why did he refer exclusively to him and not at all to Ortega? Can we take at face value his repeatedly emphasized claim that "Almān" was a priest? Is it conceivable that a Morisco came to know something that was otherwise kept secret? Or is it that he confused and amalgamated Aurel with Ortega? If so, this would suggest that he did not have their books at his disposal upon writing his treatise. We hope that future investigation, notably about Marco Aurel's background and about further copies of al-Balīshṭār's treatise, will make it possible to answer these questions.

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