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Machine learning for integrating combustion chemistry in numerical simulations

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Abstract

A strategy based on machine learning is discussed to close the gap between the detailed description of combustion chemistry and the numerical simulation of combustion systems. Indeed, the partial differential equations describing chemical kinetics are stiff and involve many degrees of freedom, making their solving in three-dimensional unsteady simulations very challenging. It is discussed in this work how a reduction of the computing cost by an order of magnitude can be achieved using a set of neural networks trained for solving chemistry. The thermochemical database used for training is composed of time evolutions of stochastic particles carrying chemical species mass fractions and temperature according to a turbulent micro-mixing problem coupled with complex chemistry. The novelty of the work lies in the decomposition of the thermochemical hyperspace into clusters to facilitate the training of neural networks. This decomposition is performed with the Kmeans algorithm, a local principal component analysis is then applied to every cluster. This new methodology for combustion chemistry reduction is tested under conditions representative of a non-premixed syngas oxy-flame.

Keywords: Combustion chemistry, Micro-mixing modeling, Principal component analysis, Artificial neural network, Chemistry reduction

1 1. Introduction

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Despite of the continuous progress in supercomputing methodologies, all the degrees of free dom required to fully describe the detail of combustion chemistry cannot be introduced in the nu-

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4 merical simulations of large-scale combustion systems, such as boilers and furnaces. Several meth5 ods have been proposed in the literature to reduce and optimize large chemical kinetic schemes,
6 such as Quasi-Steady State Approximation (QSSA) and partial equilibrium [1], Directed Rela7 tion Graph (DRG) [2], Directed Relation Graph with error propagation [3], DRG-aided sensitivity
8 analysis (DRGASA) [4] or again unimportant reaction elimination [5] (not exhaustive list).

Even after reducing the number of differential equations solved, because of the very stiff character of differential systems associated to combustion chemistry, the CPU time devoted to their
integration remains quite significant and in most cases, too large to allow for performing the multiple simulations needed to complete the quest of the optimal design for a combustion system.

These computing limitations motivated the introduction of artificial neural networks (ANN) to deal with chemistry reduction and its time integration [6] and also to develop data driven turbulent combustion modeling [7], or to analyse experimental measurements [8] and to setup digital-twins for process control [9].

Among these works, ANN and convolutional neural networks (CNN) were trained on a repre-17 sentative dataset to replace chemistry integration [10] and the method was applied to perform direct 18 numerical simulation (DNS) of a syngas oxy-flame. Nevertheless, the training of the full dataset 19 using a single or two ANNs required specific treatments, such as data augmentation through man-20 ufactured intermediate solution points for dealing with some fast reacting species [10]. These 21 operations requiring fine tuning by the user were found necessary to secure good accuracy over 22 the whole domain covering from chemically frozen-flow mixing up to the equilibrium state and 23 pollutant emissions, with fast ignition and fuel oxidation in between. 24

To avoid relying on these additional more or less ad-hoc operations, thus allowing for having a computerised and fully automatic procedure, a novel method is proposed in the present study for turbulent non-premixed flames in which techniques such as data clustering with K-means [11] and dimension reduction using LPCA [12], are combined with ANN training to secure, by construction, accuracy over the entire composition space domain (mixing, ignition, combustion and equilibrium state).

The paper is organised as follow, in the subsequent section the operating conditions considered and the novel methodology proposed are discussed. After that, the obtained ANN is tested in a posteriori manner for chemistry integration, to study the robustness of the method and the CPUtime reduction.

35 2. Chemistry integration with machine learning

36 2.1. Thermochemical conditions

The operating condition considered in this study is taken from previous works on syngas, with a chemical composition representative of those found in the steel industry in recycled exhaust gases [13]. This non-premixed syngas oxy-flame operates under a pressure of 341.3kPa, with the two inlets: (i) syngas fuel injected at 1223K for a mass flow rate of 0.74kg/s and the composition in mass fraction H₂: 0.0085, CO: 0.7852, CO₂: 0.0514, N₂: 0.1549; (ii) pure oxygen at 298K with a mass flow rate of 0.69kg/s.

43 2.2. Database generation

A database of chemical evolutions is first generated by solving for the mass fractions and the enthalpy of a set stochastic particles featuring at initial condition the concentration in the fuel or in the oxidizer inlet. Then, the species mass fraction $Y_i^p(t)$ and sensible enthalpy $h_s^p(t)$ of each stochastic particle evolve according to the following equations:

$$\frac{dY_i^{\nu}(t)}{dt} = \operatorname{MIX}_i^{p}(\tau_T) + \dot{\omega}_i^{p}, \qquad (1)$$

$$\frac{dh_s^p(t)}{dt} = \operatorname{MIX}_{h_s}^p(\tau_T) + \dot{\omega}_{h_s}^p, \qquad (2)$$

where 'MIX' denotes the stochastic turbulent micro-mixing closure for the diffusive budget in a non-premixed system. τ_T denotes the micro-mixing time and $\dot{\omega}_i^p$ and $\dot{\omega}_{h_s}^p$ are the species and enthalpy chemical sources, respectively. In the present study, the Euclidean minimum spanning tree (EMST) micro-mixing model [14] is used to model the turbulent micro-mixing.

This approach to generate thermochemical conditions representative of those observed in a real system for studying its chemical response, without explicitly solving for the flow, has been shown effective for partial oxidation of natural gas [15], aircraft engines combustion chambers [16], largescale furnaces with urea DeNOx automated control [17], micro- and meso-scale combustion systems [18] and also to build the database for machine learning application to sooting flames [19].
Here a total number of 1429 stochastic particles is employed with 740 particles (corresponding to the mass flow rate of 0.74kg/s) assigned to Fuel inlet and 689 particles (corresponding to the mass flow rate of 0.69kg/s) to O₂ inlet.



Figure 1: Clustering with K-means

From the evolution of these particles, the first step consists of identifying, from a detailed 60 chemical scheme, the important species to be retained and whose evolutions shall be learned by the 61 ANNs. This is done using the direct relation graph with error propagation method (DRGEP) [3]. 62 For the present operating conditions, this step was previously performed and reported in [10]. 63 Starting from the detailed scheme GRI3.0 [20], 11 species were identified as essential to follow 64 the dynamics of the syngas chemical evolutions: O, O₂, H, H₂, OH, HO₂, H₂O, H₂O₂, CO, CO₂ 65 and N2. A companion reduced chemistry involving 23 reactions was then optimised (see Table 2 66 of [13]). 67

⁶⁸ To save CPU time during training and to demonstrate the integration of chemistry with ANN,



Figure 2: Data clustering for parentCluster1. Temperature [K] (a) and species mass fraction [-] (b)-(j) vs H₂ mass fraction. Red: childCluster1_0. Black: childCluster1_1. Blue: childCluster1_2.

- 69 this 11-species and 23 elementary reactions scheme is used to build the time evolution dataset of
- ⁷⁰ all stochastic particles, over 4999 time-steps of $0.3\mu s$ with a mixing time $\tau_T = 0.3$ ms (a repre-
- ⁷¹ sentative value for a characteristic turbulent mixing-time as observed in shear layers for Reynolds

⁷² numbers typical of burners). N₂ was removed from the database for the pre-treatment and ANN ⁷³ learning steps. The full database thus consists of the 11 feature columns (10 species and temper-⁷⁴ ature) of the 1429 stochastic particles collected over 4999 time-steps, therefore the raw data is ⁷⁵ composed of $1429 \times 4999 = 7143571$ vectors of 10 components.

76 2.3. Data pre-treatment

To secure an appropriate analysis of the multivariate nature of the problem, the thermochemical variables obtained from the solving of Eqs. 1 and 2 are going through pre-treatments, such as centering and rescaling and unsupervised machine learning, like clustering with K-means [11], and dimension reduction with LPCA [21], before the ANN training is to be applied. This strategy is expected to close the gap between the previous works [10], in which artificial data augmentation was required to secure accuracy, and the introduction of complex chemistry in the numerical simulation of large scale furnaces.

Following the recent study by D'Alessio et al. [22] in the context of adaptive chemistry for reacting flow simulation, the original dataset is first pre-processed with centering and auto-scaling (standardizing) to insure reliable and robust results.

87 2.3.1. Data clustering

K-means [11] is a widely used and well-established unsupervised machine learning algorithm, 88 which consists in grouping similar data into different clusters. K-means is particularly suited for 89 the classification of the micro-mixing stochastic particles, which evolve through a large variety of 90 stages from chemically-frozen flow mixing, rapid ignition, fuel oxidation and combustion up to 91 the chemical equilibrium state. Moreover, the particles from Fuel and O2 inlets feature different 92 time histories, which benefit from being classified in separate clusters for chemistry ANN integra-93 tion. After clustering into separate groups, every individual subset of data has similar and regular 94 distribution to ensure effectiveness of the subsequent operations. 95

The different steps in K-means used for the thermochemical quantities (species mass fractions and temperature) are illustrated in Figure 1 and summarized as follows:

• Definition of the number of clusters.



Figure 3: Repartition of data points in childCluster1_0, childCluster1_1 and childCluster1_2. Abscissa is cluster numbering, ordinate is number of data points. Orange: particles from O_2 inlet. Blue: particles from Fuel inlet.

99	• Selecting randomly an initial cluster centroid: To ensure the quality of clustering and im-
100	prove the convergence speed, the K-means++ initialization algorithm [23] is applied. The
101	algorithm assigns randomly a first centroid, a second centroid is then chosen as farthest
102	as possible to the first centroid, a third centroid is as farthest as possible to the first two
103	centroids, and so on.

• Assigning data point to cluster with shortest Euclidean distance.

• Updating centroid locations by averaging all data points in each cluster.

Repeat the process until the centroid locations are converged by evaluating the shifting tol erance, which is chosen at 1e-20.

Defining an adequate number of clusters is crucial. The usual K-means++ initialization algorithm may lead to poor clustering in our case, where the majority of clusters concentrate in the ignition and combustion stages. To overcome this difficulty, an additional hierarchical clustering strategy was adopted; the first clustering leads to only 2 clusters (named parentCluster0 and parent-Cluster1), which separates the low and high temperature zones. The first parent containing the data of mixing, ignition and combustion stages is then clustered into 14 child clusters (childCluster0_0 to childCluster0_13). The subset of data in the second parent, representing the end of combustion followed by the equilibrium state, requires a special treatment to guarantee the accuracy of ANN training and prediction. For this reason, the second parent is classified into only 3 child clusters (childCluster1_0, childCluster1_1 and childCluster1_2), and then these 3 child clusters will have, respectively, 20, 10, 10 grandchildren (grandChild1_0_0 to grandChild1_0_19, grandChild1_1_0 to grandChild1_1_9, grandChild1_2_0 to grandChild1_2_9). Each child cluster of the first parent and each grandchild cluster of the second parent benefits from its own ANN training.

The effectiveness of K-means algorithm applied to the flame turbulence/chemistry interaction 121 (Eqs. 1 and 2) is illustrated in figures 2 and 3. Figure 2 shows how K-means automatically classifies 122 data in parentCluster1 (representing the end of combustion and the equilibrium state) into 3 distinct 123 subsets, i.e. childCluster1_0, childCluster1_1 and childCluster1_2. The childCluster1_1 contains 124 the particles from O₂ inlet, while the childCluster1_2 gathers the particles from Fuel inlet (Figure 125 3); both childClusters represent the end of combustion. The particles from O₂ and Fuel inlets 126 finally approach the equilibrium state and their data points are classified in a single common cluster 127 childCluster1_0. 128

129 2.3.2. Local principal component analysis (LPCA)

Principal component analysis (PCA), also known as a dimensional reduction technique, is used to reduce a large number of correlated variables (species mass fractions and temperature, in our case) to a smaller number of uncorrelated variables (principal components) [24]. Because of the strongly non-linear character of combustion chemistry, it is usually recommended to adopt a locally linear approach called 'local PCA' (LPCA) [21], to properly describe such dynamical system.

In our case, each cluster represents a piece-wise variation of species mass fractions and temperature during a specific combustion stage. The application of LPCA in each cluster transforms correlated variables of high variance into the most important uncorrelated principal components (PCs), which will be carefully addressed during the ANN training process. The choice of the number of PCs is essential to secure a good quality of ANN prediction, and a quite high number of PCs is usually required dealing with chemistry in reacting flows [22].

142 In practice, it was observed that reducing from 11 PCs to 6-8 PCs would still allow for keeping



Figure 4: Heat map showing correlation coefficients of data in grandChildCluster1_1_5. (a) Original variables in composition space. (b) PCs in PCA space.

more than 98% of the relevant information. However, here the full set of control variable is transformed into PC (i.e., 10 chemical species and temperature). Therefore, this does not reduce the
size of the problem to be learned, but the parametrisation of the data from the PCs considerably
eases the ANN learning process. Indeed, species and temperature are usually well-correlated, the
PCA analysis transposes these variables to non-correlated PCs and the performances of ANNs
then become much greater with these non-correlated inputs.

149 Two heat maps are presented in Figure 4 to show the correlation of original variables in com-



Figure 5: Chemistry reduction with trained ANN.

position space and of PCs in PCA space. These heat maps display the Pearson correlation coefficients, negative (respectively positive) value means that two variables vary in opposite (respectively same) directions, zero shows the absence of correlation. It is confirmed that the original variables in composition space are strongly correlated (Figure 4.a) and that PCs in PCA space are clearly uncorrelated, with practically zero correlation coefficients (Figure 4.b).

Layer	Activation function	Output shape
Input	-	(None,11)
Dense	ReLU	(None,512)
Dense	ReLU	(None,256)
Dense	ReLU	(None,128)
Dense	ReLU	(None,64)
Dense (Output)	-	(None,11)

Table 1: Structure of ANNs [10, 13]

155 2.4. Artificial neural network

The ANN regression structure (Table 1) used in this study is adopted from our previous works [10]. The ANN consists of 1 input layer, 4 dense hidden layers with the rectified linear activation function (ReLU) and 1 output layer. The total number of parameters (weights and biases) is 179851.



Figure 6: Flowchart of the a posteriori test. ('ORCh': Optimised and Reduced Chemistry.)

The ANN input is fed by the data in PCA space, composed of 11 PCs which are obtained by the projection of the 11 original variables from composition space into PCA space. The target is the increment of species composition and temperature (i.e., the ANN learns how to integrate the time evolution of chemistry). The database is split into training set (81%), validation set (9%) and



test set (10%) for model performance evaluation.

Figure 7: Time evolution of temperature [K] averaged over stochastic particles. Symbols: reference reduced scheme (Table 2 of [13]). Continuous line: Kmeans-PCA-ANN. Dashed-line: ANN without data pre-treament. Green: O_2 inlet. Red: Fuel inlet.

The training process is performed using Tensorflow 2 with GPU support (NVIDIA GeForce GTX 1080 Ti) and the Adam optimizer in default setting. To prevent the overfitting, the early stopping callback is adopted and set as 200 epochs, which means that the training will be halted after 200 epochs without improvement. Additionally, the check point callback saves the best model with the lowest mean squared error (MSE) during the training process. The total training time for all ANNs is about 11 hours.

171 **3. Results**

The methodology adopted in the study is summarised in Figure 5. To perform a posteriori tests of the model, the evolutions of the stochastic particles are recomputed replacing the integration of chemistry by the ANN, as summarised in Figure 6.

The time evolutions of temperature and species mass fractions averaged over the stochastic particles issued from one of the inlets and computed with and without ANNs, are shown in figures 77 7-9.

The case without any pre-treatment of the data (dashed-line) and a single ANN rapidly deviates from the reference solution (symbols). Without pre-treatment, the ANN captures the very first part



Figure 8: Time evolution of major species mass fractions averaged over stochastic particles. Symbols: Reference chemical scheme (Table 2 of [13]). Continuous line: Kmeans-PCA-ANN. Dashed-line: ANN without data pre-treament. Green: O_2 inlet. Red: Fuel inlet.

- of ignition, but fails to reach the equilibrium condition, see for instance the temperature response in
- 181 figure 7. (Notice that one does not make use here of the introduction of manufactured intermediate
- solutions for H_2O_2 , as discussed in [10].)



Figure 9: Time evolution of radicals and intermediate species mass fractions averaged over stochastic particles. Symbols: Reference chemical scheme (Table 2 of [13]). Continuous line: Kmeans-PCA-ANN. Dashed-line: ANN without data pre-treament.Green: O2 inlet. Red: Fuel inlet.

The solution with data pre-treatment (solid lines in figures 7-9) is in good agreement with the reference solution obtained using a CPU consuming stiff-solver for chemistry (symbols). This is the case for temperature and all species considered, even radicals and intermediate ones such as O, H, OH, HO₂ and H₂O₂. All the evolution stages (mixing, ignition, combustion and equilibrium) are well predicted.

Figure 7 shows that the ignition delay of 0.08 ms is correctly predicted by the ANN model. After the ignition, the temperature profiles feature a sharp increase to attain high values due to oxycombustion (the highest value is 3081K for the particles from the fuel side). Then, the temperature proceed towards its equilibrium state at 3037K. The trajectories of reactants (H₂, CO, O₂), and main combustion products (CO₂, H₂O) are also perfectly captured by the ANNs-based model during their evolution in all specific stages, i.e. mixing, ignition, combustion and equilibrium (Figure 8).

Regarding minor and radicals species (Figure 9), the shape and peak of the trajectories are also correctly predicted except a very slight discrepancy for H_2O_2 species departing from the oxygen inlet.

The computation time is drastically reduced by using the ANN model as given in Table 2. The simulation with ANNs (single processor calculation) is 8 times faster than the simulation for the reduced scheme, and 80 times faster than the simulation with the detailed GRI3.0 scheme. The CPU time reduction is measured here solving for micro-mixing and chemistry (Eq. (2)). This relative CPU reduction thus differs from our previous works [10], also relying on ANN for chemistry, in which the speed-up was measured solving for the aerothermochemical equations in threedimensional flows.

This quantification of the speed-up does not account for the preprocessing time needed to train the neural networks. However, the CPU effort required for training stays much smaller than the time spent performing the numerous operations required to generate a reduced chemical scheme (analysis of the most important chemical species and reaction paths and optimisation of the rate constants). What could slightly increase the CPU cost with ANN is the introduction of more degrees of freedom which may become mandatory to reproduce very specific combustion characteristics (cool flame effects, etc.).

Chemistry	Speed-up
GRI3.0 detailed scheme	1
Reduced scheme	10
ANNs (present work)	80

Table 2: Speed-up between chemistry descriptions.

212 4. Conclusion

A novel methodology based on the training of neural networks was proposed to reduce com-213 bustion chemistry and applied to a non-premixed syngas oxy-flame canonical configuration. The 214 unsupervised learning technique K-means was used to cluster the entire dataset, obtained from 215 the solution of stochastic particles in a turbulent micro-mixing system, into multiple subsets. In 216 the case of a two-inlet problem, with one of the streams bringing the energy necessary for igni-217 tion, the proposed decomposition in clusters is likely to be robust and generic. Indeed, after some 218 turbulent mixing, the mixture will always ignite to then evolve towards equilibrium. Some level 219 of finite rate chemistry may appear, but full quenching is very unlikely, especially in the case of 220 oxy-combustion studied here. However, the addition of heat losses and of non-adiabatic recircu-221 lating burnt gases may require further examination to determine the best compromise in terms of 222 clustering, still following a similar approach. 223

Local Principal Component Analysis (LPCA) was then applied on each subset to reduce the dimension of the problem and most important principal components were retained for the ANN training process. Neural networks were finally trained with the aim to replace expensive chemistry integration. The simulation results showed that the proposed method is able to predict the evolution of the thermochemical variables (temperature and species concentration) with excellent accuracy over the whole domain covering from mixing to ignition, combustion and finally chemical equilibrium. All of those with a drastically reduced simulation time.

This promising result is paving the way for the introduction of reduced chemical schemes based on ANN training in reacting flow simulations, in order to mitigate CPU times without accuracy loss.

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