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To cite this version:

HAL Id: hal-02106743
https://normandie-univ.hal.science/hal-02106743
Submitted on 23 Apr 2019

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Magnetoelectric properties of multiferroic CuCrO$_2$ studied by means of ab initio calculations and Monte Carlo simulations

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Received 22 March 2017; revised manuscript received 17 July 2017; published 25 August 2017

Motivated by the discovery of multiferroicity in the geometrically frustrated triangular antiferromagnet CuCrO$_2$ below its Néel temperature $T_N$, we investigate its magnetic and ferroelectric properties using ab initio calculations and Monte Carlo simulations. Exchange interactions up to the third nearest neighbors in the ab plane, interlayer interaction, and single ion anisotropy constants in CuCrO$_2$ are estimated by a series of density functional theory calculations. In particular, our results evidence a hard axis along the [110] direction due to the lattice distortion that takes place along this direction below $T_N$. Our Monte Carlo simulations indicate that the system possesses a Néel temperature $T_N \approx 27$ K very close to the ones reported experimentally ($T_N = 24$–26 K). Also we show that the ground state is a proper-screw magnetic configuration with an incommensurate propagation vector pointing along the [110] direction. Moreover, our work reports the emergence of spin helicity below $T_N$ which leads to ferroelectricity in the extended inverse Dzyaloshinskii-Moriya model. We confirm the electric control of spin helicity by simulating $P-E$ hysteresis loops at various temperatures.

DOI: 10.1103/PhysRevB.96.064431

I. INTRODUCTION

Through the discovery of the mineral CuFeO$_2$ in 1873, Friedel opened the door to the delafossites $ABO_2$ [1,2]. Such a family crystallizes in the layered $R3m$ space group (see Fig. 1). The diversity of properties they exhibit raises up an ever increasing interest in this class of compounds. In particular, the discovery of simultaneous transparency and $p$-type conductivity in CuAlO$_2$ by Kawazoe et al. [3], laid the groundwork for the development of transparent optoelectronic devices. Furthermore, depending on the chemical composition, a plethora of behaviors can be evidenced. For instance, for $A$ in a $d^{10}$ configuration, e.g., $A =$ Pd or Pt, highly metallic compounds with anomalous temperature dependence of the resistivity have been reported [4–7]. The transport in these compounds has been found to be strongly anisotropic, with a degree of anisotropy that may reach 1000 [4,5]. For $A$ in a $d^{10}$ configuration, the semiconducting materials CuBO$_2$, with $B =$ Cr, Fe, Rh, may be turned into promising thermoelectric ones through hole doping [9–11]—in particular, an especially high power factor has been found in the case of CuRh$_{1-x}$Mg$_x$O$_2$ [12], which transport coefficients served as a basis for the apparent fermi liquid scenario [13]. Regarding the magnetic compounds CuFeO$_2$ and CuCrO$_2$, many studies point toward a strong coupling of the magnetic and structural degrees of freedom [14–22], which paves the way to multiferroelectricity.

With its frustrated triangular lattice CuCrO$_2$ received a lot of attention since it is ferroelectric without applying magnetic fields or doping upon Cr$^{3+}$ sites, unlike CuFeO$_2$ [17,23]. The emergence of ferroelectricity in CuCrO$_2$ is induced by the proper-screw magnetic ordering below the Néel temperature $T_N$, and the control of this ferroelectricity by an applied magnetic field is very important for new spin-based device applications. CuCrO$_2$ forms a rhombohedral lattice where the edge-shared CrO$_6$ layers are alternatively stacked between Cu$^{2+}$ layers along the $c$ axis as shown in Fig. 1. Due to the weak interlayer interaction $J_{ab}$ (Fig. 2), the material behaves as a quasi-two-dimensional magnet, which makes it even more interesting.

The magnetic properties of CuCrO$_2$ have been investigated by neutron diffraction experiments [20,24–27]. It was shown that the magnetic configuration of CuCrO$_2$ below $T_N$ is proper screw with an incommensurate propagation vector $q = (0.329, 0.329, 0)$ [27] pointing along the [110] direction. Such deviation from the commensurate magnetic configuration of $q = (1/3, 1/3, 0)$ is due to the lattice distortion that takes place along the [110] direction below $T_N$ upon the spiral-spin ordering which leads to anisotropic in-plane exchange interactions $J_i$ and $J'_i$ (Fig. 2) [28]. Polarized neutron-diffraction measurements on single crystals of CuCrO$_2$ [20] showed that the spins are oriented in a spiral plane parallel to the [110] plane suggesting that the [110] direction is a hard axis.

The electric polarization emerges upon the spiral-spin ordering [20,29,30], which reflects the strong coupling between noncollinear magnetic ordering and ferroelectricity in CuCrO$_2$. Within the spin-current model or the inverse Dzyaloshinskii-Moriya (DM) mechanism [31–33], the electric polarization $P_{ij}$ produced between the canted spins $S_i$ and $S_j$, located at sites $i$ and $j$, respectively, is given by

$$P_{ij} \propto e_{ij} \times (S_i \times S_j) \equiv p_{ij},$$

where $e_{ij}$ is a unit vector joining the sites $i$ and $j$. However, Eq. (1) fails to explain the emergence of ferroelectricity in CuCrO$_2$ because in the proper-screw configurations, $(S_i \times S_j)$ is parallel to $e_{ij}$ ($e_{ij}$ is along the [110] direction due to symmetry considerations [29]) unlike the cycloid spin structures.

Based on symmetry considerations, Kaplan and Mahanti [34] introduced an additional contribution $p_{ij} \propto (S_i \times S_j)$ to the macroscopic polarization which contributes in both cycloid

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2469-9950/2017/96(6)/064431(6) 064431-1 ©2017 American Physical Society
In Sec. II we detail briefly the DFT method that we used to extract the coupling and anisotropy constants in CuCrO$_2$, while the model and MC method are presented in Sec. III. Section IV is devoted to the results where we discuss the magnetic and ferroelectric properties of CuCrO$_2$. A conclusion is given in Sec. V.

II. DFT COMPUTATIONAL METHOD

We performed a series of DFT calculations using the full-potential linear muffin-tin orbital method as implemented in RSPT [35] code. An experimental crystal structure [36] was considered, taking into account a small in-plane lattice distortion, suggested in Ref. [28]. Our results are in line with earlier calculations [21]. The DFT+$U$ [37] approach was used in order to take into account the effect of strong correlations between Cr 3$d$ electrons. The adopted values of Hubbard $U$ and Hund’s exchange $J_H$ were 2.3 and 0.96 eV, which were extracted from first-principles calculations for a similar system LiCrO$_2$ [39]. The same computational scheme was used in a prior study on the magnetic properties of CuCrO$_2$ [22]. The fully localized limit [38] form of the double-counting correction was applied. The exchange parameters between Cr$^{3+}$ ions were calculated using the linear response method [40,41]:

$$J_{ij} = \frac{T}{2S^2} \sum_n \text{Tr}_m \left[ \hat{\Delta}_i \hat{G}^{\uparrow}_{ij}(i\omega_n) \hat{\Delta}_j \hat{G}^{\downarrow}_{ji}(i\omega_n) \right],$$

where $T$ is the electronic temperature and $\omega_n$ is the $n$th Matsubara frequency, $\hat{\Delta}_i$ read the on-site exchange splitting, and $\hat{G}^{\sigma}_{ij}$ is the intersite Green’s function for the spin projection $\sigma$. The latter two quantities are matrices of $m = 2l + 1$ dimensions. The so-called muffin-tin head projection scheme was applied to construct the set of localized Cr-$d$ orbitals (more details can be found in Ref. [42]).

The $J_{ij}$’s were extracted from both ferromagnetic and antiferromagnetic configurations. The obtained values turned out to be insensitive to the assumed magnetic order, which implies that they can be used as fixed parameters in a Hamiltonian describing the interacting spins. The $J_{ij}$’s were extracted from the scalar-relativistic calculation, whereas full-relativistic treatment was employed for the estimation of the magnetocrystalline anisotropy constants. The latter were calculated directly from the total energies corresponding to three different directions of the magnetization. In order to ensure the convergence of the results, the calculations were performed on a grid of 1372 $k$ points in the entire Brillouin zone with the angular momentum cutoff $l_{\text{max}} = 8$.

III. MODEL AND MONTE CARLO SIMULATION

To model the magnetic properties of CuCrO$_2$, we note that Cr$^{3+}$ ions with $S = 3/2$ spins are large enough to be treated classically, so we used the following classical three-dimensional (3D) Heisenberg Hamiltonian:

$$H = - \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - D_x \sum_i S_{ix}^2 - D_z \sum_i S_{iz}^2 + g\mu_B \mathbf{B} \cdot \sum_i \mathbf{S}_i,$$

where $\mathbf{B}$ is the magnetic field.

FIG. 1. Delafossite structure of CuCrO$_2$.

FIG. 2. Interlayer and intralayer exchange interactions within an $ab$ plane (blue bonds correspond to $J_1$ and red bonds correspond to $J'_1$ with $J_1/J'_1 < 1$).
where \( J_{ij} \) refers to the exchange interactions up to the fourth neighbors (Fig. 2). The \( x \) axis corresponds to the [110] direction and the \( z \) axis corresponds to the [001] direction. \( D_z < 0 \) and \( D_x > 0 \) correspond to the hard and easy axes anisotropy constants, respectively. The last term corresponds to the Zeeman energy where \( \mathbf{B} \) is the applied magnetic field and \( g = 2 \) is the Landé factor.

To model the ferroelectric properties of CuCrO\(_2\) and the coupling between the spins and the electric field \( E \), we added the following term to the previous Hamiltonian:

\[
H_e = -A_0 E \cdot \sum_{\langle i,j \rangle} \mathbf{S}_i \times \mathbf{S}_j, \tag{5}
\]

where the sum runs over the magnetic bonds along the [110] direction, and \( A_0 \) is a coupling constant related to the spin-orbit and spin-exchange interactions. Adding this contribution leads to the model for multiferroics proposed by Kaplan and Mahanti [34].

Our MC simulations [43] were performed on 3D triangular lattices (Fig. 1 with only Cr\(^{3+}\) ions) with periodic boundary conditions (PBCs) using the standard Metropolis algorithm [44] and the time-step-quantified method [45] when needed.

Typically, the first \( 2 \times 10^4 \) MC steps were discarded for thermal equilibration before averaging over the next \( 3 \times 10^5 \) MC steps. Note that our results are averaged over 24 simulations with different random number sequences so that statistical fluctuations are negligible.

**IV. RESULTS AND DISCUSSIONS**

It was reported in Ref. [28] that the lattice undergoes a tiny in-plane distortion \( d = (a_2 - a_1)/a_1 \) below \( T_N \) with \( a_1 \) and \( a_2 \) being the lattice constants along the [110] and the [100] directions, respectively. As a first step, we consider \( d = 0.0001 \) [28] to calculate the exchange interactions and anisotropy constants in CuCrO\(_2\). The extracted values given in Table I (line 1) are very close to the ones reported in Ref. [46] concerning \( J_1 \) and \( J'_1 \) as well as the single ion anisotropy constants. Note that here \( J_1/J'_1 \) is very close to 1 (\( J_1/J'_1 = 0.995 \)). Knowing that PBC favors the commensurate configuration when \( J_1/J'_1 \) is close to 1, large enough sizes are required to obtain an incommensurate magnetic ground state (GS).

However, a MC simulation with \( 90 \times 90 \times 2 \) unit cells was not able to reproduce an incommensurate GS with this set of interactions \( (d = 0.0001) \). Thus larger sizes of the simulation box were required which are not accessible within reasonable computer time [47]. Therefore we enhanced the lattice distortion by a factor of 30 (i.e., \( d = 0.003 \)). We found that the new set of \( J_1/J'_1 = 0.88 \) and anisotropy constants (Table I) is a good candidate to reproduce an incommensurate GS for a system of reasonable size \( 45 \times 45 \times 2 \) unit cells. It is worth noting that the considered distortions mainly affect the first nearest neighbors interactions while the remaining interactions are not affected. Also it is very interesting to note that the magnitude of the in-plane anisotropy constant \( (D_x) \) increases when enhancing the lattice distortion reflecting that this anisotropy results from the lattice distortion.

**A. Magnetic properties**

In order to characterize the GS configuration and to estimate the Néel temperature \( T_N \) we perform a first set of simulations without applying an external magnetic field. The following procedure is considered: we start the simulations from random spin configurations at a high enough temperature \( (T_i = 35 \text{ K} > T_N) \) and we then cool down to \( T_{\text{final}} = 0.01 \text{ K} \) with a constant temperature step \( \Delta T = 0.5 \text{ K} \).

Since size effects are expected to be seen close to \( T_N \) as well as on the GS configuration in our frustrated system, our simulations were repeated for various system sizes \( (30 \times 30 \times 2, 45 \times 45 \times 2, 60 \times 60 \times 2, \text{ and } 90 \times 90 \times 2) \). No significant size dependence was seen on either the phase transition temperature or on the GS configuration (not shown here). Because further investigations like the Curie-Weiss behavior require more temperatures to be examined (\( T_i = 300 \text{ K} \)) and therefore more computer time, we choose to present here the results of the size \( 45 \times 45 \times 2 \) in order to gain time and to work with a unique size throughout the paper.

In order to estimate the Néel temperature, we calculate the specific heat per spin defined as

\[
C = \frac{1}{N} \frac{\partial U}{\partial T} = \frac{\langle E^2 \rangle_T - \langle E \rangle_T^2}{N k_B T^2}, \tag{6}
\]

where \( U(T) = \langle E \rangle_T \) with \( E \) being the energy of each magnetic configuration, \( \langle \ldots \rangle \) means thermal average, \( N \) is the number of spins, and \( k_B \) is the Boltzmann constant. For the parameter set given in Table I \((d = 0.003)\) the phase transition as signaled

![Graph](image-url)  
FIG. 3. Simulated temperature dependence of the specific heat per spin of CuCrO\(_2\). The parameter values are given in Table I for \( d = 0.003 \).
by the peak of the specific heat (Fig. 3) takes place at $T_N = 27.0 \pm 0.5$ K. This value is in good agreement with the reported experimental values ($T_N = 24–26$ K) [9,27,29]. This may be taken as a first validation of the extracted exchange interactions of Table I.

To characterize the nearly 120° GS configuration we consider the spin chirality defined as

$$\kappa_p = \frac{1}{S^2} \frac{2}{3\sqrt{3}} (S_1 \times S_2 + S_2 \times S_3 + S_3 \times S_1),$$  \hspace{1cm} (7)$$

where 1, 2, and 3 refer to the spins at the corners of each elementary triangular plaquette $p$ in an $ab$ plane. Then we define the order parameter per plane to be $\lambda = \frac{1}{n_p} \| \sum_p \kappa_p \|$ where $n_p$ is the number of plaquettes per plane, and finally the order parameter of the whole system is defined as $\kappa = \langle \lambda \rangle_T$ where $\lambda$ is the average of $\lambda$ over the $ab$ planes. We found that the direction of the vector chirality ($\lambda$) of each $ab$ plane is pointing along the [110] direction confirming the fact that the spins are oriented in the (110) plane as reported in Ref. [20]. Figure 4 shows the variation of the order parameter as a function of temperature. At $T \approx 0$ K, $\kappa \approx 0.988$ indicates a small deviation from the commensurate (120°) configuration of $\kappa = 1$. Moreover, the simulated value of $q \approx (0.322, 0.322, 0)$ confirms that the GS is an incommensurate configuration very close to the reported experimental configuration of $q = (0.329, 0.329, 0)$ [27]. This good agreement may be taken as further validation of the parameters of Table I.

On the other hand, the magnetic field dependence of the magnetization calculated along the easy axis ($z$ axis) shows a linear behavior ($-5T < H_z < 5T$) confirming the antiferromagnetic nature of the GS (not shown here).
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FIG. 8. P-E hysteresis loops simulated at different temperatures in CuCrO2.

Magnetic properties under 0.5 T were simulated between 300 and 2 K to estimate the Curie-Weiss temperature (θCW). Figure 5 shows the variation of the magnetization and inverse susceptibility measured along the applied magnetic field. It can be seen that 1/χ obeys well the Curie-Weiss law for antiferromagnets [1/χ = (T + θCW)/C, with C the Curie constant] at high temperatures with θCW = 175 ± 1 K close to the measured experimental values (θCW = 160–170 K) [9,48]. The 1/χ curve starts to deviate from the linear behavior at about 100 K. In order to understand the origin of this deviation we calculated the temperature dependence of the spin-spin correlation function defined as G(rij,T) = ⟨S_i·S_j⟩T along the [110] direction. As shown in Fig. 6, short-range antiferromagnetic correlations start to develop below ∼100 K, which leads to the deviation from the Curie-Weiss law seen in Fig. 5. Furthermore, these correlation functions exhibit inflection points close to TN estimated from the specific heat curve (Fig. 3). Besides, an anomaly in the magnetization curve (Fig. 5) appears at 28 ± 2 K consistent with the estimate of TN from the specific heat curve. We note that the ratio θCW/TN ≈ 6.5 (<1) reflects the frustrated nature of the GS [49,50].

B. Ferroelectric properties

In this section, we consider the Hamiltonian H + Hr. In these simulations, we apply a poling electric field during the cooling process to obtain a single ferroelectric domain. We then turn it off just before statistical averaging to calculate p2 which is associated to the spontaneous ferroelectric polarization [Eq. (2)] according to Ref. [34]. Figure 7 shows the temperature dependence of P110, the projection of p2 along the [110] direction, which starts to develop at TN. It is clearly seen that by switching the poling electric field, P110 can be reversed.

Further insight into the degree of electrical polarization may be gained through the knowledge of the P-E hysteresis loops, which are shown in Fig. 8 at different temperatures. P110 shows a linear E dependence without hysteresis above TN because the system is in the paraelectric phase, while clear hysteresis loops are seen for temperatures below TN. This strongly suggests that ferroelectricity is induced by the out-of-plane incommensurate magnetic configuration, in agreement with Ref. [30].

Also, it can be seen that below TN the saturation field E_{sat} ≈ 8.9 × 10^{-2} MV/m is independent of the temperature. The hysteresis loop simulated at 5 K shows an electric coercive field for P110 reversal Er ≈ 4.2 × 10^{-2} MV/m very close to that measured experimentally (Er = 5.1 × 10^{-2} MV/m) [51]. Note that the reversal of P110 results from the reversal of the helicity of each ab atomic plane. Thus our simulations confirm the electric control of spin helicity in CuCrO2 as reported in Ref. [20].

V. CONCLUSION

In this paper, we proposed estimates of the exchange interactions and single ion anisotropy constants in the multiferroic CuCrO2 using DFT calculations. They were checked against the experimental Néel and Curie-Weiss temperatures as well as the electric coercive field, thereby proving them to be good candidates to model the magnetoelectric properties of CuCrO2. We showed that the lattice distortion that takes place below TN is responsible for the appearance of a weak in-plane hard-axis anisotropy. Regarding the magnetic properties, we obtained a peak in the specific heat curve at TN ≈ 27 K very close to the experimental observations. Furthermore the ground state has been shown to be an antiferromagnetic incommensurate proper-screw configuration. The estimated θCW ≈ 175 K is in a good agreement with experimental data too. Also, our simulated P-E hysteresis loops confirm the electric control of spin helicity which is related to the ferroelectric polarization below TN.

ACKNOWLEDGMENTS

We gratefully thank M. Alouani and S. Hébert for stimulating discussions. We are grateful to the Centre Régional Informatique et d’Applications Numériques de Normandie (CRIANN) where our simulations were performed as Project No. 2015004. We also acknowledge the computational resources provided by the Swedish National Infrastructure for Computing (SNIC) and Uppsala Multidisciplinary Center for Advanced Computational Science (UPPMAX). The authors acknowledge the financial support of the French Agence Nationale de la Recherche, Grant No. ANR-10-LABX-09-01 and the Centre national de la recherche scientifique through Labex EMC3.


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